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THE UNAIDED EYE

PART I

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SYNOPSIS

Introduction, Historical; Principal Parts of Eye, Lachrymal Fluid, Cornea, Astigmatism; Aqueous and Vitreous Humours; Construction of Crystalline Lens, Refractive Indices of Parts; Elimination of Star Image Defects; Hevelius-Hooke Controversy; Accommodation; Ciliary Processes; Zonule of Zinn; Intra-Ocular Pressure; The Exit Pupil; Light-gathering Factors; Iris Pupil sizes; Speed of Movement; Relationship of Right and Left Pupils; Variation of Pupil with Variable Illumination of Macula Lutea and several Zones; Law of Variation; Reflex Control of Iris; Chart of Accommodation; Variation of Pupil Size with Accommodation; Law of Variation.

THE function of an optician is to provide optical aids to the eye. It is desirable, therefore, to know what are its capabilities. This involves some preliminary knowledge of the physiology of those parts that concern the dioptric working and the reception of images by the retina.

For the physiological details I am more particularly indebted to Helmholtz's *Physiologische Optik* and to Luciani's *Human Physiology*, Volume IV, which deals with the sensory organs, and Volume III, which deals with the muscular and nervous systems.

An optician, and more particularly a manufacturing optician, may be justified in claiming that of all the sensory organs the eye is the most remarkable. Helmholtz, when exposing its so-called defects, was careful to express his great admiration of an organ which so completely fulfils the functions for which it was created.

Recently it has become practicable by means of the Optophone to substitute to a limited but important extent the ear for the eye as the organ of appreciation.

It will soon be possible for the blind to read the daily papers and printed works of any kind. To do so, a special series of transformations is necessary. Light reflected from the white portions between the printed letters is caused to act upon a selenium cell and vary its electrical resistance. From light to electricity is thus the first transformation. The second, from electricity to sound, is more easily effected by means of a telephone. It is with the eye, however, which from the earliest times has received the greatest consideration, that we opticians are concerned.

Pythagoras (about 580 B.C. to 500 B.C.) believed that vision was due to the passage of particles from the object to the eye and, although no authentic records of Pythagoras himself exist, the Pythagorian School taught that colour was not something that existed outside our eyes, nor yet inside them, but some effect that arose from the various disturbances which the object viewed gave rise to in the eye.

About one hundred years later Empedocles, who flourished about the year 440 B.C., taught that vision was due to something corporeal that issued from the eye and passed to the object, whence it was reflected back to the eye.

Aristotle (384 B.C. to 322 B.C.) differed from Empedocles in teaching that light was incorporeal. These diverse views formed the subject of strenuous discussion for many hundreds of years, but from the *Optical Treasury* of Alhazen it would appear that about the eleventh century the eye was generally recognised as purely a receiving instrument for the light and a transmitter to the brain.

Although no definite date can be assigned to Alhazen's work, it is known that he died at Cairo in the year 1038 A.D. Abou-Aly-Al-Hazan-Ben-Alhazen, whose name is spelt in a great variety of ways, was a native of Bessarabia. Having conceived a scheme for the Control of the Nile, he was commissioned by the Khalif to survey the site. When Alhazen viewed the Nile he realised the futility of his project and the personal danger the acknowledgment of his failure would involve. He accordingly feigned madness and became sane again only after the death of the Khalif. During this period of feigned insanity he compiled a number of scientific works—such an occupation being evidently considered as not inconsistent with his malady! Fortunately one of these works was a treatise on optics which was translated into Latin and published by Risner in 1572 under the title of *Alhazen Opticae Thesaurus*. This work is indeed a treasury of optics.

René Descartes' conception of the human eye was almost modern in its detail and in order to describe the eye I shall first quote Descartes' description translated from the work entitled *Renati Descartes Opera Philosophica* dated 1656. Chapter III of the section on dioptrics reads as follows:

> If it were possible to cut the eye across a plane through the pupil in such a manner that none of the contained liquors could escape or any of the parts become displaced, the section would appear as represented in the diagram, Fig. 1.
>
> *ABCB* is a membrane somewhat dense and hard, having the form of a vase which serves as a receptacle for all the interior parts. *DEF* is a thinner membrane which extends like a curtain over the inner surface of the former. *ZH* is what is commonly termed the optic nerve. It consists of a vast number of small fibres, the extremities of which are spread over the whole surface *GHI*, where they are mixed with innumerable fine veins

and arteries, thus forming a species of very tender flesh which appears as a third membrane covering all the base of the second. *K*, *L* and *M* are three very pellucid liquors which distend all these membranes and have the particular forms indicated in the diagram.

Experiments have taught me that the medium *L*, which is called the crystalline humour, produces nearly the same refraction as glass or crystal, and the two remaining humours a somewhat smaller refraction nearly equal to that of water*....

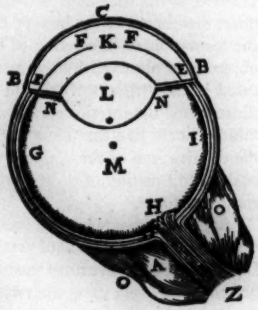

Fig. 1.

In the first membrane the part *BCB* is transparent and more curved (humped) than the remainder *BAB*. In the second the interior surface of the part *EE* which faces the base of the eye is all black and opaque and has at the centre of its anterior portion a small round hole called the pupil which appears very black when viewed from without.

But this aperture is not always open to the same extent. The part *EE* of the second membrane in which it lies floats freely in the very liquid humour *K*. It has the appearance of a slight muscle which extends or contracts† according as the objects which are viewed are near or far away or are more or less strongly illuminated, or when it is the wish to view them more or less distinctly....

EN, *EN* are numerous black filaments embracing on all sides the humour *L*. They originate in the second membrane at the place where the third ends and evidently form a species of minute tendons by means of which this humour, through becoming more curved or flatter according as it is desired to observe near or more distant objects, alters somewhat the whole figure of the body of the eye. This may be proved by experiment. For, if while looking intently at a distant tower or mountain a book is placed at a short distance before the eyes, none of the letters will be seen except indistinctly until the configuration of the parts alters slightly.

Finally *O* and *O* are six or seven external muscles attached to the eye by means of which it may be turned in all directions and incidentally also by pressure or retraction they may alter the figure.

It is interesting to observe from the above account that Descartes associated the act of accommodation of the eye not only with a change of form of the crystalline humour *L* but also with a change in the configuration of the body of the eye itself. He assumed that the contraction of the tendons *EN* would not only modify the shape of the crystalline lens but would also reduce the diameter of the eyeball and thus increase the axial length of the eye through the compression of the internal humours.

* The paragraph here omitted is inconsistent with the present day undulatory theory of the propagation of light.

† Extension or contraction of the muscles should correspond with reduction or enlargement respectively of the pupil. Probably Descartes refers here to the radial muscles of the iris, not to the sphincter.

That Descartes, like his predecessor Father Scheiner, was familiar with the dioptric mechanism of the eye is evident from Fig. 2, which is reproduced from the fifth chapter of his work. The newly excised eye was inserted in the roof *Z*, *Z* of a dark chamber *P* and the outer membrane at the base of the eye being removed the formation on the retina of the images of external objects was observed by the interested patriarch indicated in the diagram. It should be noted that in both illustrations the interior surface is shown covered with minute fibres like the pile of velvet, these fibres being the terminations of the optic nerves, and Descartes discusses the function of these fibres in the visual appreciation of form.

Fig. 2.

A modern diagrammatic section of the human eye showing a greater amount of detail is indicated in Fig. 3. Light entering the eye must traverse the following media before it impinges upon the retina:

(1) A thin liquid lachrymal film which covers the front of the cornea and keeps it moist;

(2) The cornea which acts as a lens and serves as the window of the eye;

(3) The aqueous humour filling the front or anterior chamber of the eye;

(4) The front surface of the sac containing the crystalline lens;

(5) The various concentric layers of the crystalline lens;

(6) The posterior surface of the sac containing the lens;

(7) The vitreous humour;

(8) The hyaloid membrane;

(9) The various layers of the retina.

All these media, including portions of the last-mentioned, are very transparent to light.

Over the surface of the cornea there is maintained a film of liquid supplied by the lachrymal gland and distributed by the inner surface of the eyelids in the involuntary process of winking. This surface tension film ensures the transparency of the entrance surface of the eye. Considering the thinness of the film under normal conditions and the fact that the refractive index of the liquid differs but little from that of the cornea, which is 1·377, it does not appear that the liquid has any special dioptric function.

The cornea, 1, Fig. 3, consists of a horny transparent dome-shaped plate whose thickness is very nearly uniform over the portion traversed by the incident light. It is terminated at both surfaces by a layer of epithelial cells.

Measurements of the curvature of the refracting surfaces are usually made on the living eye by means of the ophthalmometer, originally devised by Helmholtz, as post-mortem measurements are unreliable. It is not easy by existing means to obtain accurate results and in any case the data vary considerably in different

persons. Thus the thickness of the cornea at the centre, as recorded by different observers, varies from 0·5 to 1·4 mm. The refractive index is about 1·38 and the radius of curvature of the anterior surface is about 7 to 8 mm., but the curvature is by no means equal in all meridians and in consequence there may be important

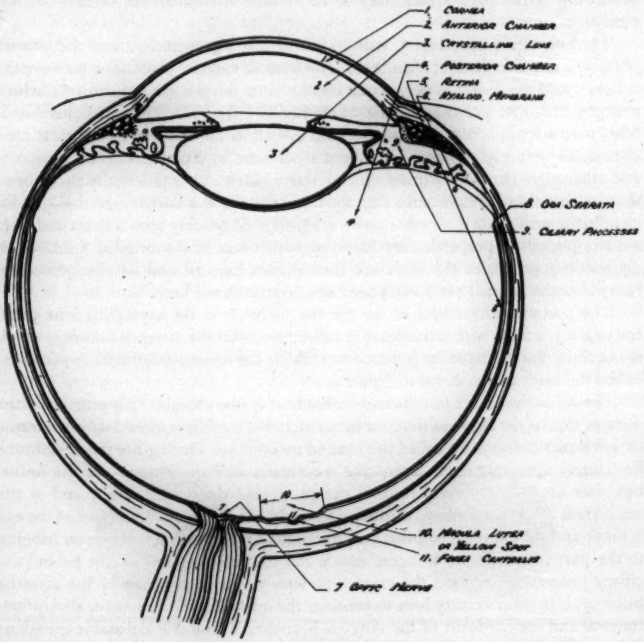

Fig. 3.

differences in the focal length of the corneal surface in different planes, thus giving rise to astigmatism, which may be regular or irregular, according to the disposition of the maximum and minimum planes of refraction.

In the great majority of astigmatic eyes the refraction in the more vertical meridian is the stronger. Compared with the front surface the inner or posterior surface of the cornea is comparatively unimportant so far as the effect of irregularities of curvature are concerned. At the front surface the refraction is from air whose index is practically unity to a medium having an index of 1·38, that is, a difference of refractive index of 0·38, whereas at the posterior surface the refraction

is from the cornea to the aqueous humour, the refractive index difference being less than 0·05.

Dioptrically it is the front surface of the cornea that is of particular importance. The astigmatism of the eye is partly attributable to the crystalline lens and to slight decentring of the parts which may be of variable amount under certain circumstances.

The crystalline lens, Fig. 3, with its suspensory ligament separates the interior of the eye into two distinct chambers. The front or anterior chamber 2 between the cornea 1 and the lens 3 is very much smaller than the posterior chamber 4, which occupies about 80 per cent. of the whole body of the eye. The front chamber is filled with a limpid, very transparent fluid known as the aqueous humour. It consists almost entirely of water with about 1 per cent. to 2 per cent. of alkaline salts and other ingredients in solution. Its refractive index 1·34 is practically the same as that of water and differs only slightly from that of the cornea. If the anterior chamber is artificially drained, a new supply of fluid quickly accumulates and after a short period its properties are identical with those of the original fluid. Such debatable questions as the source of the aqueous humour and its absorption and renewal under normal conditions need not be considered here.

The posterior chamber 4 of the eye that is between the crystalline lens 3 and the retina 5 is filled with a transparent substance called the vitreous humour, which is not fluid like the aqueous humour but which has a semi-gelatinous appearance. When the mass is cut, drops of liquid exude.

The refractive index of the vitreous humour in the chamber 4 is practically the same as that of the aqueous humour in the chamber 2. It is enclosed within a transparent structureless sac 6 called the hyaloid membrane. During life this membrane lies closely upon the inner layer—the membrana limitans interna—of the retina, but after death it separates readily, except around the optic nerve 7 and at the ora serrata 8. At these places it adheres strongly. If an equatorial section of the eye is made and the contents emptied out, the hyaloid membrane 6 will remain hanging to the part where the optic nerve enters and to the ora serrata 8, just behind the ciliary processes 9 around the crystalline lens 3. As in the case of the aqueous humour, it is not necessary here to consider the questions of the source, absorption, renewal and nourishment of the vitreous humour. These still debatable questions do not directly concern the optician.

As it is not the intention to deal with the receiving mechanism of the eye—that is, the retina—in this lecture, it remains only to describe the crystalline lens and its accessories.

A camera is focussed by moving the whole lens towards or from the receiving plate. In the eye the power is divided into a larger constant portion and a smaller variable portion, the former being almost entirely due to the first surface of the cornea, and the latter to the crystalline lens. This lens is formed of a material which, if available in practice, would prove invaluable to the designer of optical instruments. It possesses certain peculiar elastic properties which enable it to be deformed in such a way as to alter its power and to return to its original shape when

the force is removed. It has the useful feature of having a refractive index which is not constant throughout but which varies from a maximum at the centre to a minimum at the periphery.

The substance of the lens appears to be arranged in layers around a somewhat harder core. At the centre the refractive index is 1·411, at the intermediate layers 1·406, at the periphery 1·388, and the index of the capsule is 1·360. Helmholtz states that "the substance of the lens is doubly refracting. When it is placed between crossed Nicol prisms there are visible the dark cross and coloured rings exhibited by a uniaxial crystal cut normal to the optical axis." From the centre towards the periphery the index difference increases rapidly.

Owing to the greater curvature of the central layers combined with the greater index, the power of the central portion is considerably greater than that of the outer, and when focussed for infinity the equivalent index of refraction of the lens considered as a homogeneous whole, namely about 1·42, is actually greater than that of the nucleus. As was previously stated, it is not easy to measure accurately the curvature of the lens in the living eye. Helmholtz in the construction of his schematic eye assumed the front surface of the lens, that is, facing the entering light, to have a radius of curvature of 10 mm. and the posterior surface a radius of 6 mm. The front surface is therefore actually nearly twice as flat as the posterior surface when the eye is focussed for infinity.

Although the crystalline lens is a highly transparent substance, it is not structureless but appears to be built up of cells arranged in layers around an almost spherical core, and grouped in a complicated stellate form around the axis, as indicated in Figs. 4 and 5, which represent the front surface and the rear surface of the lens

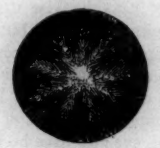

Fig. 4.

Fig. 5.

respectively. According to Bowman the substance of the lens in the neighbourhood of the central planes separating the sections is distinctly different in character from the remainder of the lens and this suggests a difference in refractive index.

If a point source of light such as a small hole in a plate brightly illuminated from behind is viewed by the unaided eye, the hole will not be seen distinctly. It will appear irregularly star-shaped. A row of holes will appear as a row of stars, all oriented the same way. Rotation of the hole will not affect the orientation, which, however, alters when the head is inclined sideways. The star appearance corresponds with the stellate structure of the lens and may be due to reflection of light at scraping incidence at the interfaces of different refractive indices between the

various sectors of the lens*. When the illumination is greatly reduced the stray light disappears in the case of some observers.

It will be observed that in the centre of the lens there is a portion which appears to be more homogeneous and which actually is so. If a diaphragm having a hole about 1 mm. diameter is placed in front of the eye so as to obscure the whole of the stellate portion of the crystalline lens, leaving only the clear central portion unobstructed, the brightly illuminated hole which previously appeared to be star-shaped will now appear clear and distinct and in its true form. No difficulty need be experienced in centring the eye opposite the hole because want of centring causes the image to appear indistinct. The hole must not be so small as to introduce diffraction images that appreciably affect the appearance. This sighting hole device has proved very useful in the experiments to be described later for determining the accuracy with which the unaided eye is able to resolve two point sources. It should be repeated that only rays at scraping incidence are reflected from the interfaces. If the point source is examined by means of a telescope the proportion of the rays that enter the eye at scraping incidence is small and the point source does not appear star-shaped. Similarly in the case of a white spot visible by scattered light, the intensity of any stray light reflected from the interfaces of the crystalline lens is by contrast too small to be appreciated by the retina.

In this connection it is interesting to recall the famous Hevelius-Hooke controversy. Dr Hooke advocated the use of collimator sights on astronomical instruments, whereas Hevelius preferred open sights with which he attained remarkably accurate results. Dr Hooke asserted that such accuracy was beyond the power of the unaided eye. It appears from the writings of Hevelius, who was born at Danzig in 1611 and died on his birthday in 1687, that, when observing, he placed a narrow vertical slit about one millimetre wide in front of his otherwise unaided eye. Through such a slit a source of light appears quite distinct at the sides but prolonged starwise in a direction parallel to the slot. These prolongations become curved towards one side or the other as the eye is moved across the slot and thus there is provided a simple indication as to when the eye is central. Now, in the case of Hevelius, it is quite possible that in addition to exceptional keenness of vision one of the diametral interfaces in the crystalline lens of his eye may have been arranged vertically and accordingly parallel to the sighting slot and it may have been exceptionally wide. With his direct sights a star would thus appear to him clearly defined and better suited, therefore, for accurate measurements. According to Montucla in his *History of Mathematics*, Volume II, the astronomer Halley paid a special visit to the observatory of Hevelius in order to test the accuracy of the observations made there. Halley used apparatus fitted with telescope sights while Hevelius used apparatus fitted with his open sights. The agreement of the two sets of observations was satisfactory. It is interesting to note that, although Hevelius preferred open sights, his observatory also contained instruments fitted with telescope sights.

* Helmholtz ascribes the stellate appearance to reflections from the irregular edge of the pupil.

Ophthalmometric observations show that in the process of accommodation the anterior surface of the crystalline lens becomes more curved as the object approaches the eye and that over the greater part of the posterior surface facing the retina there is practically no change of curvature. As the anterior surface becomes more curved, the axial thickness of the lens increases while the equatorial diameter decreases. It is improbable that any change of volume of the lens itself is involved. The crystalline lens rests upon the vitreous humour from which it is separated by the posterior sac of the lens. This membrane which separates the aqueous humour entirely from the vitreous humour and thus the anterior chamber from the posterior, is practically an extension from the ora serrata of the hyaloid membrane which encloses the vitreous humour. The membrane is lettered 6 in Fig. 6, which shows the crystalline lens 3 and the meridional, radial, and circular muscles, 10, 11 and 12, that operate it in the act of focussing or accommodation.

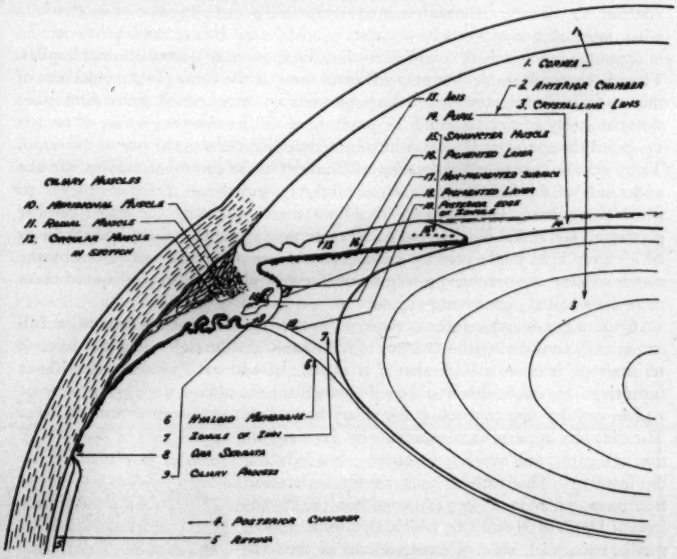

Fig. 6.

It is generally agreed that the alterations of the crystalline lens are due to the action of the ciliary muscles, 10, 11 and 12, operating through the intermediary of the ciliary processes 9 and the suspensory ligaments or zonule of Zinn, 7, attached to the anterior and equatorial surface of the sac containing the lens. There appear

to be three sets of ciliary muscles, all of which play some part in the act of accommodation. The meridional muscles, numbered 10 in the illustration, as the name implies, lie in a meridional section of the eye. The circular muscles numbered 12 lie in a plane normal to the axis of the eye and concentric with it. The radial muscles, 11, occupy an intermediate position between these two sets, which they appear to bind together. From the ring of ciliary muscles about seventy projections called the ciliary processes, 9, extend radially inwards towards the crystalline lens to which they are connected through the intermediary of the zonule of Zinn, 7.

Now, although it is evident that all three sets of muscles take part in the process of accommodation, it is not clear what their separate functions are. Considered generally, if the circular muscles 12 relax, the diameter of the ring carrying the ciliary processes will increase and exercise an equatorial pull on the crystalline lens that will tend to flatten its anterior surface. Conversely, if the circular muscles 12 contract, the diameter diminishes and the crystalline lens by reason of its elasticity when thus released recovers its previous curved form. It is certain that the action of accommodation is more complicated than the above very general account implies. The iris 13, for example, not only advances towards the cornea as the thickness of the crystalline lens increases, but the pupil 14 or aperture near its centre diminishes simultaneously in size. Around the pupil there will be observed a ring of muscle 15, called the sphincter muscle, which by contraction reduces the size of the pupil. There are also certain radial muscles which assist in the control of the iris. On the under side of the iris there is a dense black-pigmented non-transparent layer 16 which prevents any light from entering the eye except through the pupil. Similar pigmented layers will be observed on all those portions of the ciliary body through which stray light might possibly pass. The portions 17 actually embraced by the zonule of Zinn are not thus protected but it will be observed that in front of them there are situated pigmented portions 18.

Quite a considerable force is required to flatten the crystalline lens to its full extent and, considering the fineness of the membrane through which the force is transmitted, it is remarkable that it is capable of withstanding the pull without rupturing. No doubt this would be the case if the membrane were in the form of a plain annular ring connecting the ciliary body direct to the capsule of the lens. The difficulty appears to be overcome by the distribution of the total pull over a large surface, thus reducing the intensity. This will be more readily understood from an examination of Fig. 7 extracted from the *Physiologische Optik* of Helmholtz, which represents a side, that is, equatorial, view of one quadrant of the lens. The vertical line *a*, *b* is the axis of the lens. The connection of the posterior surface of the capsule 6 in Fig. 6 is represented by the line *c*, *d*. Since the force of compression of the lens is resisted by the vitreous humour the tension in this membrane during the process of accommodation is probably negligible. The anterior membrane which is called the zonule of Zinn is

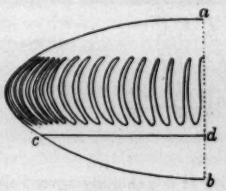

Fig. 7.

folded somewhat like the ruffles worn around the neck by Elizabethan courtiers. Its point of attachment to the lens capsule is indicated by the wavy line of the illustration. To the anterior surface between the folds, of which there are approximately seventy, there are attached the separate ciliary processes 9, Fig. 6. Thus it will be seen that by this ingenious arrangement the pull is distributed over a large area and to the lens through a very great length of contact. In Fig. 6 the bottom of the fold is indicated by the line 19.

Between the folded line and the line *c*, *d* in Fig. 7, that is, between the zonule of Zinn and the posterior membrane, there is a space called the canal of Petit. This space is in communication with the anterior chamber of the eye through slits provided in the zonule and the space is accordingly filled with the aqueous humour. Thus the whole space from the cornea to the membrane covering the vitreous humour is filled with the same liquid.

It should be observed that the iris only rests upon the crystalline lens at its central extremity, that is, around the pupil, and that the aqueous humour has access behind it and thence to the canal of Petit.

Although the volume of the crystalline lens does not alter during the act of accommodation, its change of form involves an alteration in the relative sizes of the portions of the anterior chamber in front of and behind the iris respectively. Since the total volume of the chamber remains unchanged or practically so, there must be a flow of liquid through the pupil from the one side of the iris to the other. Helmholtz, however, considered it improbable that such a displacement could take place as rapidly as the action of accommodation requires, since the edge of the iris rests upon the lens, and he suggested that the adjustment was effected by a suitable movement of the ciliary parts around the base of the iris.

In altering the form of the crystalline lens the ciliary muscles exercise a considerable force which must be resisted by that portion of the wall of the eye to which the muscles are attached. But as the wall of the eye is not rigid, it would tend to yield under the pull of the ciliary muscles. Such a reduction of the equatorial diameter of the eye would involve a corresponding increase in the axial length since the eye is filled completely by the various humours, as was considered to be the case by Descartes. But it is accepted that no such changes of form occur. The reason is that inside the eye there is a pressure equivalent to from 20 to 30 millimetres of mercury. This pressure keeps the walls of the eye rigidly distended and enables them to resist the pull of the ciliary muscles. It is accordingly of the greatest importance in the dioptric function of the eye. Descartes' assumption previously referred to, that accommodation involved an axial variation of length, suggests that he was unaware of this intra-ocular pressure. The reason why there is liquid in the anterior chamber will now be evident. Not only does it enable the crystalline lens to alter its form freely but it also, on account of its pressure, preserves the form of the walls of the eye and particularly the portion to which the retina is attached.

Probably the portion of the eye that most directly interests the designer of optical instruments is the pupil. Suppose the optician is asked to design a telescope

having the maximum illumination at a magnification of ten diameters. He knows that the exit pupil of the instrument need not be much greater than the pupil of the eye which limits the amount of light that can reach the retina. It should be noted, however, that a larger exit pupil has the advantage that the eye need not be held so steadily before the eyepiece. Generally, however, this feature does not justify the increased cost and increased size of the instrument involved in such an increase of the exit pupil. If the diameter of the pupil and hence the maximum useful diameter of the exit pupil of the instrument is known, the diameter of the telescope objective is found by multiplying the pupil diameter by the magnifying power. Thus, suppose the exit pupil is 0·25 inch, then for a magnifying power of ten the objective daylight diameter must be $10 \times 0{\cdot}25$ inch, or 2·5 inches. For a doublet objective of this diameter, the choice of an aperture factor, that is, the ratio of the focal length to the diameter, of 7, will not involve the computer in any difficulties. Therefore the focal length should be $2{\cdot}5 \times 7 = 17{\cdot}5$ inches. Thus when the exit pupil and magnification are known, the objective diameter is also known and a suitable focal length can then be deduced. It is desirable, therefore, to know what is the diameter of the pupil of the eye under various conditions.

The optician is not much concerned with the extreme diameter of the pupil which is only reached when the eye is kept in complete darkness for an hour or more. Instantaneous flash-light photographs of the dark-accommodated eye have indicated a maximum pupil diameter of slightly over 0·3 inch or 8 mm. As the movement of the iris is comparatively slow, the diameter cannot become appreciably reduced during the small time of the exposure.

The author's eye takes about 3 seconds to close from a diameter of 0·25 inch to 0·08 inch. There does not appear to be any appreciable difference in the time of opening and of closing through the same amount.

The pupil cannot close completely. Its minimum diameter is about 0·08 inch or 2 mm. Thus the range of action of the pupil is from 0·3 inch to 0·08 inch, or from 8 to 2 mm. These are the apparent sizes of the pupil, as viewed through the cornea and aqueous humour. The actual sizes are about 7 per cent. less.

The iris cannot protect the eye from light sufficiently strong to give rise to painful sensations or to damage the retina. Its action is too slow and dignified. It can only adjust the intensity of the light to suit the best conditions of vision. If the eye is subjected to an abnormally intense light, the eyelids come instantly into action. Their rapidity of action is very much greater than that of the iris. If the light is very intense the eyelids are compressed, thus increasing their thickness and opacity. At the same time the head is bent downwards and the arm is raised in front of the eyes to afford additional protection. Like the reflex movement of the eyelids these actions are instinctive.

If the dark-accommodated eye opens out to 0·3 inch (8 mm.), it will be evident that the eye when applied to an optical instrument will open out to a less extent and that the amount will depend upon the quantity of light proceeding from the exit pupil.

In German specifications of low power instruments, such as binoculars of

6 diameter magnification, it was customary to specify an exit pupil of 5 to 6 mm. (0·2 to 0·24 inch) and 7 to 8 mm. in night binoculars. In high power optical instruments such as range-finders having a magnification of 25 to 30 diameters, an exit pupil of from 3 to 2·5 mm. was frequently specified. On the assumption that the area of the exit pupil is a measure of the quantity of light proceeding from the instrument, the light-gathering power or the illumination factor was expressed by the Germans in terms of the square of the pupil diameter in millimetres and this practice was followed in Russia and other Continental countries. Thus the light-gathering power of an instrument having an exit pupil of 6 mm. diameter was denoted by the number 36.

Such an assumption, however, is quite wrong and very misleading. The quantity of the emergent light is proportional not only to the area of the exit pupil but also to the intensity of the emergent light. The exit pupils of some instruments appear much darker than others, the amount depending upon the proportion of the light lost in passing through the optical system. Thus in a Galileo field glass the percentage of entering light that emerges may be about 80 per cent., whereas in a prism binocular it may only be 60 per cent. A more approximate comparison of the light-gathering capacities of different instruments of the same magnifying power might be obtained from the quotient of the exit pupil diameter and the number of glass air transmission surfaces in the respective instruments. In testing the light-gathering powers of competitive instruments of the same type, it is desirable to measure the exit pupil diameters and the intensity of the emergent light.

It will be of interest now to consider what is the behaviour of the iris when the eye looks into an optical instrument because it is these conditions that the designer has to consider. Supposing the exit pupil diameter is 0·1 inch, the apparent pupil of the eye would probably open out to 0·1 inch. It would not open beyond that amount even if the quantity of light were reduced*. If the exit pupil were 0·5 inch diameter, the apparent iris pupil would open out to an extent determined by the quantity of light but, in the case of an optical instrument of the simplest description used in daylight, it would probably not open out to a much greater diameter than 0·27 inch. Only under exceptional circumstances, such as during night observations of indistinct objects, would the pupil expand to 0·3 inch.

Most of the apparent pupil diameters referred to in this paper were measured by means of a simple pupilometer which consisted of strips of glass on which were photographed circular black opaque spots of diameters varying from 0·05 inch to 0·35 inch, by steps of 0·01 inch. This strip was placed as close to the cornea as possible and was so arranged that it could be rapidly moved from spot to spot. As in such an experiment it is essential to keep the eye central with the exit pupil, the head was fixed in position by means of a plaster cast of the brow and in some cases by adjustable rests. All external light other than that proceeding from the exit pupil was of course carefully excluded. The angular field and magnification were kept small so as to ensure a small obliquity of the rays entering the eye, but from

* This statement was not confirmed by subsequent measurements taken under other conditions or by instantaneous flashlight photographs, reference to which will be made in Part III.

the optical data a correction factor was derived by means of which the results were corrected to compensate for the small obliquity of the extreme rays and the distance of the occulting spot in front of the cornea. It will be understood, as previously stated, that the results give the apparent pupil diameters, which on account of the corneal refraction are larger than the actual by approximately 7 per cent. It is with the apparent diameters that the optician is concerned.

The results obtained with this simple pupilometer were checked and found to be in fair agreement with those obtained by means of more elaborate apparatus in which the image of the occulting spot was projected into the plane of the iris and in which accordingly it was unnecessary to make any correction for the obliquity of the rays, however great.

Suppose the exit pupil of the instrument were 0·2 inch diameter and that of the

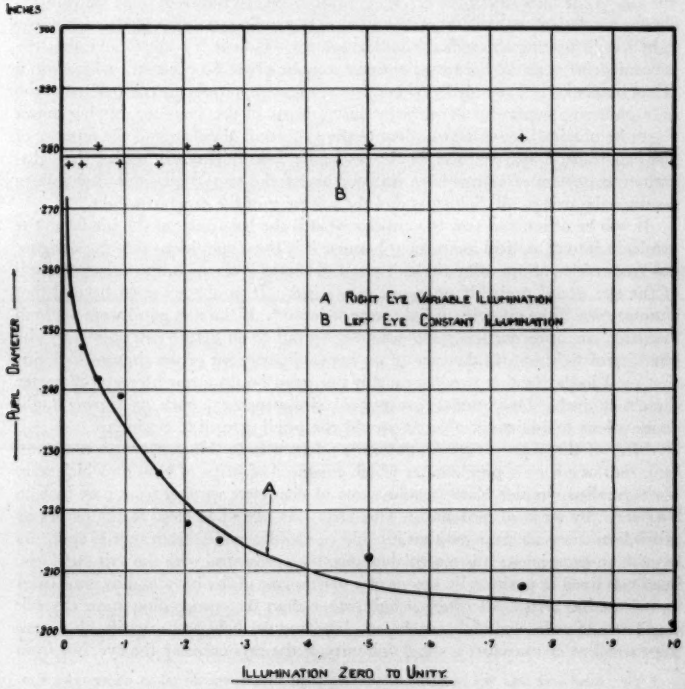

Fig. 8.

eye 0·1 inch. If a stop 0·1 inch were inserted, the observer would see his own iris open out slowly beyond the diameter of the spot. Viewed in this manner the motion of the iris is very deliberate. It resembles somewhat the unfolding of a flower at sunrise, as depicted by the cinematograph. Since the iris movement is comparatively slow, it is easy to insert the appropriate stop before the iris has time to expand by an important amount.

It is desirable now to consider the effect on the pupil of variations of the illumination. For this purpose a telescope having an exit pupil of 0·5 inch diameter, that is, much larger than the maximum pupil diameter of the eye, was constructed. The telescope was directed towards a collimator illuminated by a Pointolite lamp, the intensity of which was varied by a flicker wheel. In the check experiments, which were generally confirmatory, the variation of the intensity was effected by altering the distance of the source of light.

Preliminary experiments were carried out to test the relationship of the two eyes. It is well known that, as regards directional motions, the two eyes move together and their convergence upon a near or distant object is quite instinctive. Are the pupils interconnected in the same manner? If light is projected into the one eye, does it cause the pupil of the other eye to contract? In carrying out such a test it is of the greatest importance to ensure that no stray light from the source illuminating the one eye reaches the other. From Fig. 8 it will be evident that the pupils may act independently.

Before the right eye there was placed the telescope system above described, provided with the flicker wheel for the variation of the illumination. Before the left eye a telescope illuminated by a constant diffuse source of light of low intensity was arranged.

Ordinates in the diagram indicate the variation of the iris pupil diameter, and abscissae the variation of the light from about zero to unity. Although the right iris pupil varied in diameter over a considerable range, as indicated by the curve *A*, the left iris pupil diameter remained constant, as indicated by the horizontal line *B*. It is not surprising that the pupils should act independently of each other, considering that the sensitivity of the retinas of the two eyes are often different.

Fig. 9 indicates the variation of the pupils of the left eyes of two different observers as the illumination was varied by means of the flicker wheel. The curves are arranged differently from the right eye curve, that is, curve *A* of the previous diagram, Fig. 8. In diagram 9 the abscissae now represent the logarithms of the percentage illumination and the ordinates the logarithms of the corresponding pupil areas. The illumination was varied from 5 per cent. to 100 per cent. and the pupil diameters varied from 0·25 to 0·20 inch in the case of one observer, and from 0·23 to 0·18 inch in the case of the other.

When plotted logarithmically, as in the diagram, it will be seen that the curve is practically a straight line over a considerable length. At the bottom (large pupil diameter) end, the curve is straight and it is reasonable to assume that, if the light had been further reduced, the pupil would have opened still further, if sufficient time had been allowed. At the top (small pupil diameter) end, that is, at the high

illumination end, the curve deviates from the straight line, notwithstanding that the illumination was only sufficiently great to close down the pupil to a diameter of about 0·18 inch, whereas the minimum diameter is about 0·08 inch. From the diagram it will be evident that the straight portion of both curves can be represented

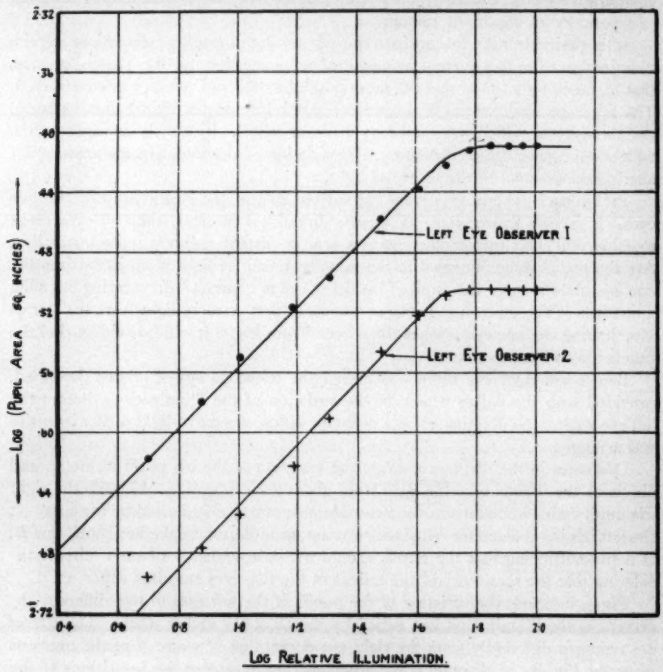

Fig. 9.

by the equation Area $= K$ (Illumination)^x and from a measurement of the slope it appears that in the case of both observers the value of $x = \frac{1}{5}$. Therefore

$$\text{Area of Pupil} = K\,(\text{Illumination})^{\frac{1}{5}}.$$

If, then, the illumination is increased thirty-two-fold, the pupil would contract to half its area.

Fig. 10 shows the results obtained from the tests of the right eyes of the same two observers under the previous conditions. For the same variation of the light

the pupil of the one observer varied from 0·26 to 0·20 inch and the other from 0·24 to 0·19 inch. The value of x as measured from the straight line portions of the curves is identical in both cases, namely $\frac{1}{5\cdot2}$. Thus, for both right eyes,

$$\text{Area} = K\,(\text{Illumination})^{\frac{1}{5\cdot2}}.$$

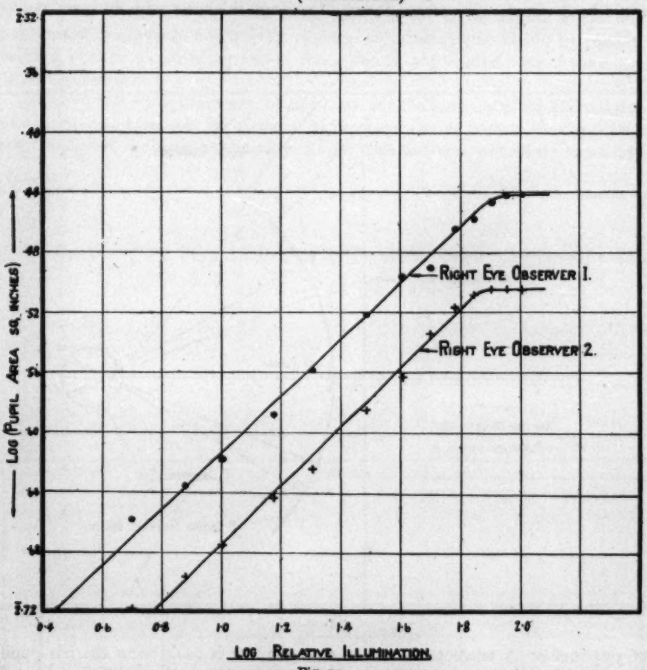

Fig. 10.

In order to reduce the pupil area to one-half it would be necessary to increase the illumination thirty-seven-fold. Thus in these two observers, each of whom was practically normal as regards both eyes, the sensitiveness of the left eye to illumination so far as the iris reflexes are concerned was slightly greater than that of the right. This is possibly due to fatigue of the right eye through greater use.

In all four tests the angular field was about 5° and therefore the four curves indicate the sensitiveness of the macula lutea or yellow spot of the retina upon which alone the light fell.

Similar data for various zones of the retina, concentric with the macula lutea, were obtained by means of the apparatus indicated in Fig. 11. Light from a "Pointolite" lamp was received upon a diffusing screen *A* from which it passed to a portion of a spherical glass screen *B*, one surface of which was fine-ground. When viewed by the observer's eye *C*, no variation of the illumination over the screen *B* could be detected. The screen *B* was of uniform thickness with an inner radius

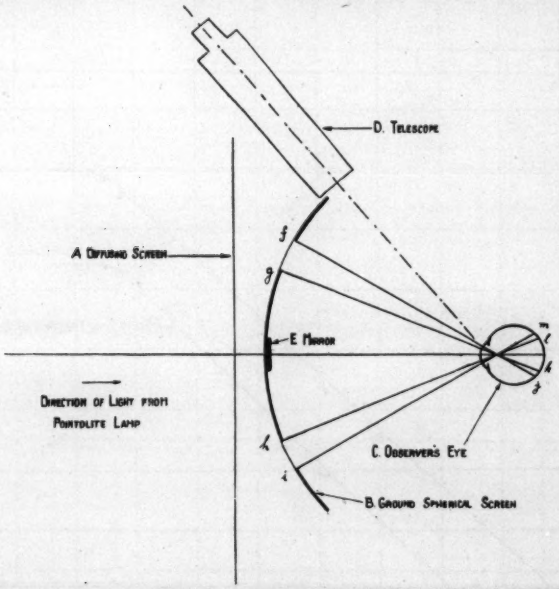

Fig. 11.

of 3·75 inches. A telescope *D* was so arranged that it could view the iris pupil and measure its vertical diameter upon a scale in the eyepiece. The amount of light falling upon the iris was in all cases sufficient to make it visible by means of the telescope. Immediately below the pupil there was arranged a horizontal scale which together with the pupil could be seen reflected in the small mirror *E* by the observer *C* himself, who was thus able in many of the experiments to obtain check measurements of the pupil diameter. The main purpose of the telescope *D* was to ensure that the position of the eye *C* relatively to the mirror *E* was not altered. Any changes were easily detected by the observer at the telescope *D*. The central portion of the screen from (*g*) to (*h*) was painted black as well as the portion beyond

(*f*) and (*i*), thus leaving a luminous ring or zone whose section is indicated by the lines (*f*, *g*) and (*h*, *i*). On the retina of the eye *C* there would then be formed a corresponding image of the zone whose section is indicated by the lines (*j*, *k*) and (*l*, *m*). Substituting Helmholtz's schematic eye for the eye *C* of the observer, the widths of a series of retinal zones all of the same area were calculated and, knowing the position of the eye and the radius of the screen *B*, the widths and positions of the corresponding series of zones on *B* were determined. By suitably painting out the requisite portions of the screen *B*, any desired zone of the retina could be illuminated.

Since all the retinal zones were of equal area and the intensity of the illumination was maintained constant, the diameter of the pupil should remain constant if the sensitiveness of the retina to quantity of light were constant over all the zones. If,

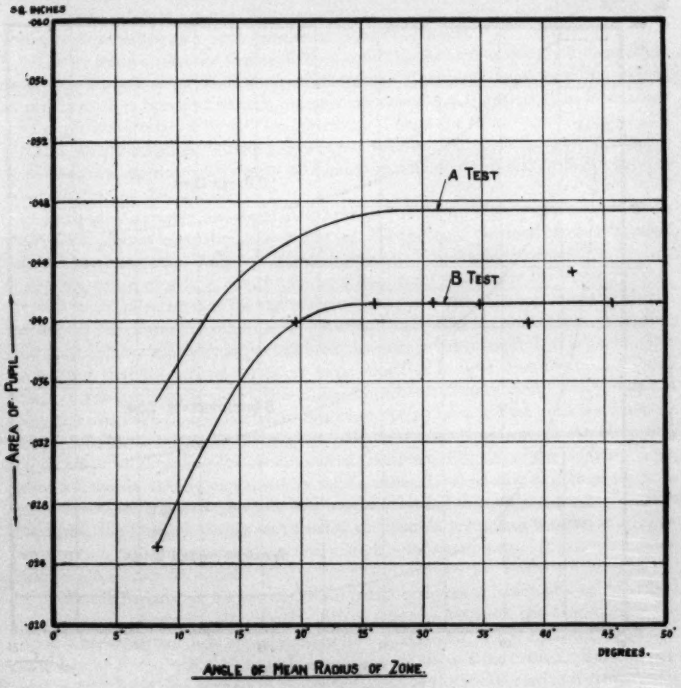

Fig. 12.

however, the retina were less sensitive over any particular zone, the pupil would expand so as to increase the quantity reaching each point and accordingly increase the total quantity of light distributed over the zone.

Fig. 12 shows the results of two tests. The area of each zone of the retina was 0·06 square inches. Abscissae represent the angle subtended by the mean radius of the zone at the centre of curvature of the retina, not the nodal point of the Helmholtz schematic eye, and ordinates the area of the iris pupil. It will be seen that, as the distance of the zone from the macula lutea increases, the sensitiveness of the retina to light, so far as the iris reflexes are concerned, rapidly diminishes, and that at an angular distance of about 25° the sensitiveness has reached a minimum value. In the case of curve *B* it will be observed that the iris ceased to respond when the pupil diameter was considerably less than its maximum. From this and the previous

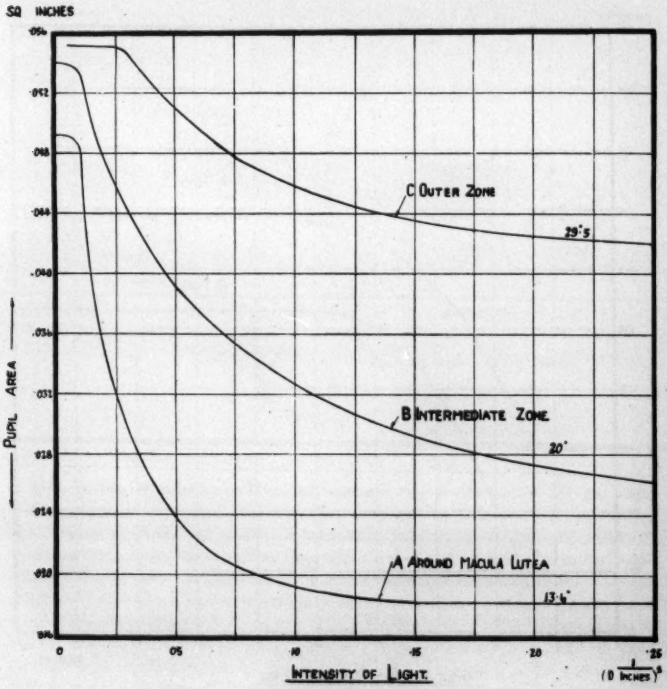

Fig. 13.

Figs. 9 and 10 it would appear that it is the macula lutea and a comparatively limited portion of the retina around it that control the variations of the pupil diameter.

By means of the same apparatus the variation of the pupil area, as the intensity of the light reaching each of the retinal zones was varied, was determined by altering the distance from the screen of the "Pointolite" source and measuring the diameter of the pupil as before. As the intensity of the light varies inversely as the square of the distance D of the source, the abscissae in Fig. 13, which are proportional to $D^{-\frac{1}{2}}$, represent intensities of illumination. As before, the ordinates represent the pupil areas. Curve A indicates the pupil variations corresponding with the variable excitation of the zone immediately surrounding the macula lutea or yellow spot. The mean angular distance of the zone from the fovea centralis was 13·6°, as indicated on the diagram. This zone is particularly sensitive to variations of the light and evidently exercises an important control over the pupil. As the intensity is increased, that is, as the illuminating source approaches, the pupil diameter rapidly diminishes and soon reaches a minimum value.

Curve B for a zone 20° from the fovea centralis shows that already the sensitiveness has become diminished very considerably. The pupil diameter diminishes at a much smaller rate and reaches a minimum value that is greater than in the case of the previous zone. Curve C for a zone 29·5° from the fovea centralis shows still greater insensitiveness. This portion of the retina requires all the light it can get and the pupil opens out to nearly its maximum diameter and responds but little to the variation of the intensity.

As in the corresponding diagram for the macula lutea, Figs. 9 and 10, the curves have been plotted logarithmically in Fig. 14. The straight portions can be expressed by the same equation Area $= K$ (Illumination)^x. For the zone nearest the macula lutea the value of x is $\frac{1}{3}$, for the intermediate zone $\frac{1}{4}$, and for the outer zone $\frac{1}{5}$. It will be recollected that for the macula lutea the value of x was about $\frac{1}{2}$, that is to say, so far as the iris reflexes are concerned the zones immediately surrounding the macula lutea are more important than the macula lutea itself. If it is permissible to assume that the central zones are more sensitive to light than the macula lutea itself or the peripheral zones, the above-mentioned results might help to explain why the existence of an indistinct object but not its form is best detected when its image falls just outside the macula lutea. Faint distant objects at sea can often first be detected by directing the eye towards a position a little above the horizon. This phenomenon is usually explained by the fact that the existence of a large object is more easily appreciated than that of a minute one and that on account of defective focussing the image is larger just outside the macula lutea than it is inside it. The assumption that increased reflex activity implies increased mental appreciation must not be too readily accepted.

When light falls on the retina certain peripheral organs which will be described in a subsequent lecture are excited. Afferent nerves transmit the sensation to the brain where they are duly appreciated as form and colour by the mind which decides whether or not it will take further action and, if so, of what nature. But the sensation of light is also carried to the cervical end of the spinal cord and sympathetic

nervous system, which automatically responds and transmits through the ganglion cells and efferent nerves the appropriate orders to the peripheral organs of, for example, the iris sphincter muscle, which on contraction reduces the pupil diameter.

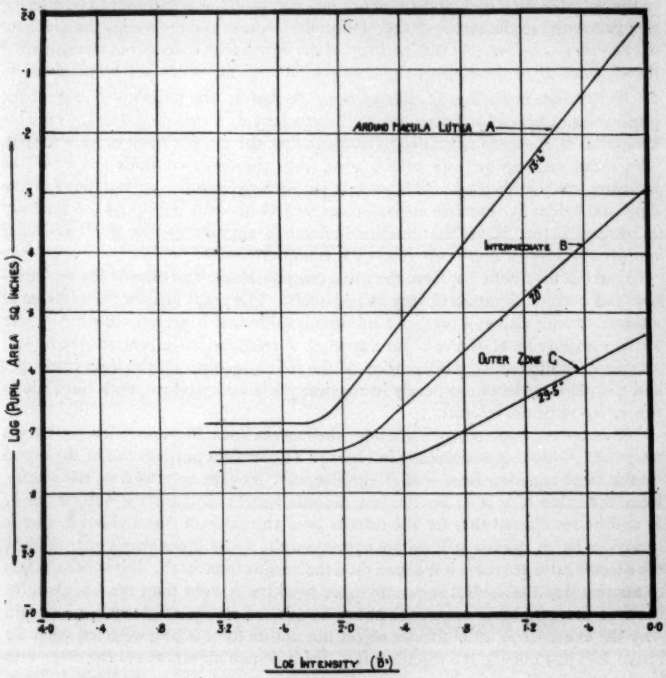

Fig. 14.

When the brain receives a message some particular connection has to be made in order to transmit an appropriate order to the appropriate locus determined by the brain. The action is voluntary; from the transmitting organ through the spinal cord and sympathetic system to the receiving organ in, for example, the iris sphincter muscle, the circuit is permanent and the reflex action is involuntary. The same message in general calls forth the same kind of response.

There is another type of reflex controlling the pupil diameter which depends upon the state of accommodation of the eye. If the focus of the crystalline lens were invariable the image of an object as it approached the eye would proceed to positions

beyond the retina of the normal eye, where it would be out of focus. At long distances the change would be small. At short distances the change of position would alter with increasing rapidity. To keep the image on the retina and therefore in focus the power of the crystalline lens is automatically varied by reflex action which presumably depends on the size of the circles of confusion of the images of point sources on the retina. The change of power is due to an alteration of the radius of curvature of the anterior surface of the lens. Practically no change of the posterior surface facing the retina takes place. As the object approaches the eye, the anterior surface becomes more curved and the axial thickness of the lens increases. Since there is no observable change in the volume, the equatorial diameter decreases and accordingly there is a very considerable change in the curvature of the peripheral zones of the lens.

Simultaneously with the increase of power and thickness of the lens the iris necessarily advances towards the cornea but at the same time the diameter of the pupil contracts, thus cutting off a larger portion of the rim of the lens. These changes are indicated in Fig. 15, the full lines of which show the arrangement and form of the parts when the eye is accommodated for infinity and the dotted lines the arrangement when accommodated for a near object.

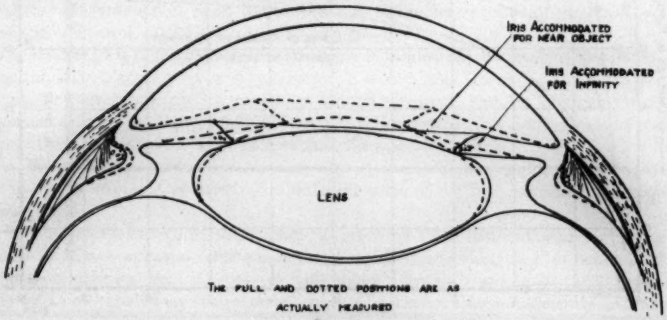
IRIS ACCOMMODATED FOR NEAR OBJECT
IRIS ACCOMMODATED FOR INFINITY
LENS
THE FULL AND DOTTED POSITIONS ARE AS ACTUALLY MEASURED

Fig. 15.

To determine the manner in which the pupil diameter varies during accommodation, an object the illumination of which was maintained constant was made to approach the eye and the pupil diameter was measured for each position of the object. From the results plotted in curve I, Fig. 16, it will be seen that no variation of the pupil takes place until the object is about 12 inches from the eye and that thereafter the contraction is very rapid. From a comparison with Fig. 13 it will be evident that the variation of the pupil during accommodation is not comparable with that due to a variation of the illumination which in any case was kept constant.

On the basis of the schematic eye, the focal lengths of the crystalline lens corresponding with the positions of the object were calculated. They are indicated

by curve II. From the values of the focal lengths the corresponding radii of curvature of the anterior surface as indicated by curve III were calculated.

It will be seen that the curve I does not closely correspond with either the focus

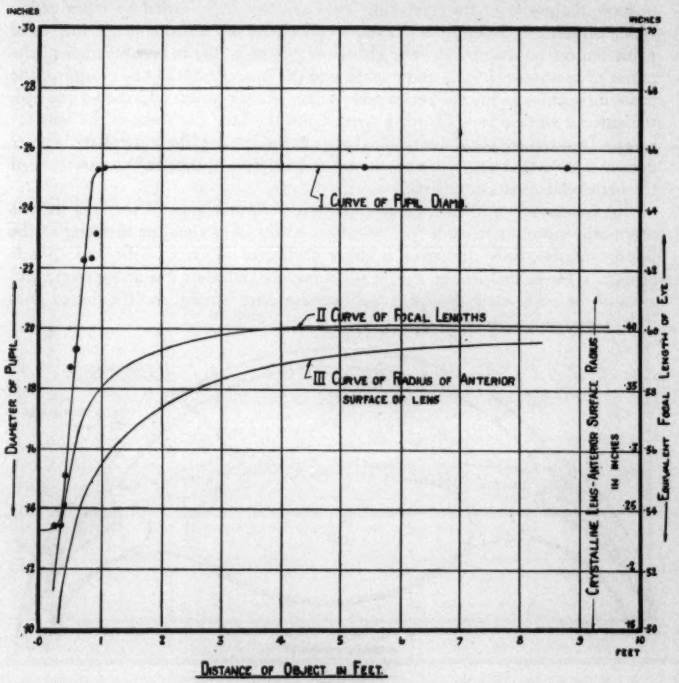

Fig. 16.

or radius curve. Probably the adjustment of the pupil is determined by the amount of the spherical aberration which would result from the passage of the light through the highly curved peripheral zones. Thus the same type of reflex action that controls the ciliary muscles and alters the focus of the crystalline lens when the circles of confusion are too great may also control the sphincter muscle of the iris and by reducing the pupil stop out the portion of the lens that is giving rise to excessive spherical aberration.

DISCUSSION

Mr W. A. Dixey said that in the experiments described the time of exposure was a very important factor. When one comes out of the dark into a bright light an appreciable amount of time elapses before the pupil of the eye adapts itself to the new conditions, whereas in the matter of accommodation the pupil acts almost instantaneously. He thought that, if the author had given his time values, some reason for these differences might have been discovered.

He said it was difficult to accept the statement that the size of the pupil of the eye was affected by the exit pupil of the instrument which was being used.

He hoped that in the second part of his paper Mr French would deal with some of the problems connected with binocular vision, the conditions which control variations in stereoscopic vision, the relation between convergence and accommodation, and the way in which the coordination of these two functions is affected by abnormality in either of them.

Mr F. E. Lamplough considered that, as the behaviour of the eye is the result of centuries of evolution, one would expect the pupil from habit to open out widely when the illumination of an instrument was feeble, irrespective of the smallness of the exit pupil.

With regard to the change from darkness to moderately bright light, the illumination at first appears to be very intense. It would be of interest to know whether the eye therefore at first contracts to its fullest extent, and, on becoming more accustomed to the light, again dilates. If so this would be a marked case of the pupil size varying with other factors than the intensity of light.

In the case of certain instruments with eyepieces of large field, the rolling of the eye as attention is directed to different parts of the field necessitates a large exit pupil.

The pupil size, when fully dilated, as given by the author indicates that in the case of an astronomical telescope having an objective working at $F/15$, when used on faint nebulae, etc., a power of only about 3 per inch should be used, i.e. an eyepiece of 5 inches equivalent focal length. This is not always sufficiently realised.

Mr L. C. Martin did not think that the flicker arrangement for varying intensities was satisfactory, for, although the apparent intensity of illumination is proportional to the sector aperture, it is not definitely known whether the muscular reflex action in the eye is proportional to the same quantity. It would have been better to have used some other of the well-known methods of varying intensity.

Mr P. F. Everitt did not agree with the statement that the star-shaped appearance of an illuminated pin-hole was not observed when viewed in a telescope, unless the author was probably referring to a high power telescope.

Mr C. L. Redding referred to the Helmholtz and Tscherning theories of accommodation. He called attention to the difference between the structure of the circular portion of the ciliary muscle and the other circular muscles of the eye and eyelid, such as the sphincter of the iris or the orbicularis; under the microscope

the fibres forming the circular portion of the ciliary muscle seemed to be very similar in character to the rest of the ciliary. The whole question of accommodation was open to discussion.

Mr H. S. Ryland pointed out that it is not always an advantage to have the exit pupil of an instrument larger than the average pupillary diameter. In the case of variable power telescopes, at high power the exit pupil is about 5 mm. diameter and 10 or 15 at low power; while the definition at high power is good, that at low power falls off badly unless the observer's eye is exactly on the principal axis of the instrument. Also at low power an apparent parallax will exist. A distinct improvement is effected by gearing an iris diaphragm with the change power medium to keep the exit pupil diameter constant.

With regard to light transmission Mr Ryland suggested that instruments should be numbered by using the square of the exit pupil diameter multiplied by the measured transmission value.

Prof. F. J. Cheshire said that the paper was the result of an important investigation which was being carried out by Mr French with regard to the question of the magnifying powers of range-finders. It was rather remarkable that in coincidence range-finders an accuracy of less than 1 second of angle could be obtained.

The result obtained by the author that there is no sympathetic reaction between the eyes was in his opinion opposed to the result of the simple experiment performed by all students of optics, viz. that of looking at a bright object with one eye and then uncovering the other.

With regard to the question of spherical aberration he drew attention to the fact that in the last edition of Helmholtz's book which had been practically re-written by Gullstrand it is stated that the central zone of the eye of 4 mm. radius brings the rays to the same focus as the paraxial rays.

Mr S. D. Chalmers thought that the question of the relative sizes of the pupil of the eye and of an instrument had not been quite clearly understood. The author probably meant that the pupil of the eye will not open further than is useful.

He agreed with Mr Martin's remarks about the flicker method. Mr French, however, had in some of his experiments used the method of varying the distance of the light source.

With regard to accommodation it was difficult to believe that the amount obtainable in a young child could be explained solely by the steepening of the anterior surface of the lens. Neither Helmholtz's nor Tscherning's theory accounts for the magnitude of the accommodation in this case. This was probably due to the fact that measurements of accommodation have been primarily made on the eyes of adults.

The author communicated the following reply:

Throughout the various experiments of the lecture to which the members have so kindly given their consideration the conditions were made similar to those existing in the ordinary use of optical instruments rather than under abnormal conditions. Extremes were avoided as far as possible. The illumination of the

room in which the experiments were conducted was subdued, blinds being drawn over the windows. But the apparatus, including the head of the operator, was thoroughly enclosed to avoid all stray light from affecting the results. Thus whenever the observer after a series of observations withdrew from the apparatus his eye was exposed only to subdued light. No attempt was made to make the eye extremely dark adapted, such a condition being of more interest to the astronomer than to the ordinary optical engineer.

The programme which Mr Dixey has outlined is an extremely wide one. To deal exhaustively with even the simplest features of the eye is somewhat difficult within the limits of a lecture. Indeed I fear there has been some misunderstanding of certain of the statements owing to the inevitable omission of qualifications.

In dealing with the variation of the pupil diameter with the quantity of light and with accommodation for the object distance it was not intended to suggest that there are not several other conditions that produce similar results. Anyone who conducts such experiments must be impressed with the variety and complexity of the conditions. As Professor Cheshire remarked there are well-known circumstances which imply an interconnection of the pupils of the two eyes. In the first instance I attempted to make use of the fact mentioned to control one pupil by varying the light entering the other, but the experiment failed because of the ease with which the one pupil can be maintained constant while the other is varied through a wide range, as indicated in the diagram. It was this point I wished to emphasise.

In reply to Mr Lamplough's remarks, I personally do not know of any ordinary telescope in which it is necessary to roll the eye in order to view the whole field. An apparent angular field of 50° to 60° is considered very great. In a submarine periscope of 1·5 diams. magnification the angular field is 40°, that is an apparent field of 60°. When the eye point is satisfactory, the eye can view the whole field without any rolling movement. To provide for a larger factor than 60° is difficult and angular fields beyond 40° involve silvered reflecting surfaces owing to the marginal rays being beyond the critical angle.

Mr Martin's remarks regarding flicker methods of observation were quite well realised. In the experiments other methods of varying the light were also employed and the results appeared to be in fair agreement and within the general limits of accuracy.

In reply to Mr Everitt if the magnification and field of the telescope are so low that a large part of the emergent light traverses the planes of the crystalline lens at scraping incidence then I should expect a star appearance of a point source object.

In reply to Mr Redding I am not a physiologist and am therefore not able to discuss the details of the various accommodation theories. Such investigations as I have made have only served to show me how very complex the subject is. Like Mr Redding I cannot say I have ever been able to trace definitely the circular fibres of the ciliary muscle.

To provide an exit pupil larger than that of the eye, as Mr Ryland desires, is only commercially practicable in low power instruments. For an exit pupil diameter

of 0·3 inch and a magnification of 30 diameters the objective diameter would be 9 inches.

I understand that in the numerical designation of light powers of instruments Mr Ryland wishes loss of light to be taken into account, as I suggest. But then that is not what is done in Germany. Under the German method two binoculars of the same exit pupil diameter would have the same light power number although the prisms of one might be made of ordinary plate glass while those of the other were made of highly transparent optical glass. The remarks of Professor Cheshire and Mr Chalmers do not I think call for any special reply.

THE SPACING OF GLASS-WORKING TOOLS

By T. SMITH, B.A.

(From the National Physical Laboratory.)

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A QUESTION of interest to the manufacturing optician which does not appear to have received the attention its importance warrants is the difference that should exist in the curvatures of neighbouring tools in a set intended for general work. With a properly arranged series a wide variety of miscellaneous lenses can undoubtedly be more economically produced than with a less well proportioned set, and it is to the interest of a manufacturer to arrange any additions to his stock as far as possible in accordance with a predetermined principle so as to secure the greatest degree of usefulness from them. It will be at once conceded that a tool need not have the exact radius given by calculations, since between some limits which may vary with the radius of the tool and the particular lens for which it is required any change in the radius does not produce defects which can be detected by even very sensitive tests, and between certain wider limits the consequent defects, though evident and measurable, may yet be permitted under the conditions in which the instrument is to be used. A series of tools so arranged that the separation of successive members is comparable with the latter limits for the most exacting forms of instrument is the ideal at which to aim.

As an illustration of the economy in the number of tools which may be effected by fixing the exact values of the curvatures according to a predetermined plan, suppose that in the neighbourhood of the radius r it has been found that the proper spacing is in accordance with an arithmetical progression with an interval 6ρ. Let the actual values for four successive tools on a suitable system be $r + 9\rho$, $r + 3\rho$, $r - 3\rho$, $r - 9\rho$. It may be assumed that no tool exists in the neighbourhood of r—say nothing nearer than $r \pm 2\rho$. In calculating out a new lens it is found that a radius of $r + 2\rho$ is required, and later that a radius of nearly $r + 6\rho$ is wanted. If a maker worked on the system he would make the first tool of radius $r + 3\rho$, and it would afterwards be found just near enough to the $r + 6\rho$ to be usable. If, on the other hand, he had no system the first radius would be $r + 2\rho$, and the interval between this and $r + 6\rho$ being 4ρ, while the greatest admissible departure from the calculated value is 3ρ, a second tool would be required. Later it might happen that a radius of $r - 2\rho$ would be desired, involving in any case a new tool. If this were made on the system of radius $r - 3\rho$ it would be available for any radius down to $r - 6\rho$, whereas without the system a fourth tool would be necessary for the latter radius. Thus it may very well happen in a number of cases that the adoption of a system will enable two tools to serve instead of four.

Various systems for spacing tools have been suggested, but they do not appear to be applicable over more than a portion of the range to be covered. For example

there are very obvious advantages in a system of tools whose curvatures are in geometrical progression, but this would undoubtedly prove a wasteful system for shallow tools if the common ratio were suitably arranged for more moderate curvatures. It appears desirable therefore to consider the uses to which tools of different sizes will be put and to take these into consideration when seeking a suitable principle on which a series should be built.

It may be assumed that the systems to be constructed will extend from the smallest microscope objectives to telescope objectives of moderate diameters, such as 200 mm. The construction of very large astronomical objectives is so exceptional that they may be left out of consideration. It is doubtful whether any useful purpose would be served by an attempt to deal systematically with tools for such work. Thus flat tools will be required for many purposes, but there will be no necessity for any tool between the flat and some radius expressible in metres—say of the order of magnitude of ten metres. Very little consideration is required to show that at this end of the range the spacing should be approximately an arithmetical progression of curvatures.

Between these very shallow tools and the deep ones at the other end of the range lie the great bulk of the tools, and it is in this region that a proper system of spacing will yield the greatest advantages. Over this region critical comparison between the spacing of the tools that have been required by manufacturers in a period of several years and those determined by any theory is of importance both in estimating the reasonableness of the premises on which the theory rests and in finding suitable values for any general constants that may occur in the resulting expressions for the radius of a tool.

At the other end of the scale there will be a definite minimum value for the radius determined by the size of the front lens of the smallest immersion objective for which any demand will be made. In this region of extremely deep surfaces it is to be expected that the distribution of the tools of any manufacturer will be decidedly irregular. Variations in glass meltings may on the one hand involve a close cluster of tools for one special surface, and on the other hand the radical changes customary in the general character of the objective as the focal length is altered may result in the presence of comparatively large gaps in the curvatures of successive tools. The maker's tool list will not therefore be necessarily of any very great value towards this extreme end of the range over which tools are required. It appears therefore that chief consideration should be given to the central region, though reference to the extreme regions may be of value for ancillary purposes.

The exact character of the law which it is desired to determine will depend upon the extreme demands made from the optical systems to be constructed. The typical systems of the region to be investigated are photographic lenses and telescope objectives of small to moderately large sizes. The important features to be kept in mind are the variations in focal length and in aperture. The two change together, but not in proportion, the focal length increasing relatively faster than the aperture Considering only systems which reach a predetermined degree of accuracy in the image, it will be found possible to express by means of some equation the minimum

value the focal length f may have for a given aperture x. A simple relation which may be considered is

$$af = x(x + b) \qquad \ldots\ldots(1),$$

where a and b are two constant lengths. According to this equation the relative aperture falls continuously from the limit a/b for very small lenses, and would tend to zero for lenses of extremely large aperture if such lenses could be made. The necessity for this latter result should be particularly noted. It might be thought that the limit for large values of x should be some constant other than zero. It must however be remembered that a definite criterion for the quality of the definition is in question, say a maximum departure of a given fraction of a wave length from a sphere in the emergent wave front. Now in similar systems any linear error will be proportional to the focal length, and therefore for very large focal lengths the attainment of this constant standard implies the tendency of the relative aperture towards zero.

It is next necessary to consider the dependence of the limiting curvatures of the surfaces upon the aperture. As the aperture is decreased in size from moderate values to very small values, the ratio of the shortest radius that may be employed to the aperture will tend to some definite minimum limit, and on the other hand as the aperture is increased to large values the ratio will tend to an upper limit. The variation over the whole range will be expressible by some equation of which a simple form that may be considered is

$$\frac{r}{x} = k \cdot \frac{x + p}{x + q} \qquad \ldots\ldots(2).$$

Equations (1) and (2) are only examples of the relations that may express the extreme demands which the manufacturer has to face, but they are probably capable of fitting the requirements so far as they are known with sufficient accuracy, and such forms will be utilised in the following discussion.

In a similar way a sufficiently correct expression may be obtained for the variation of the maximum field of view with the aperture. This will not be required in the ensuing discussion and such an equation is not therefore formulated.

Now assume that a formula has been obtained for a lens with spherical surfaces which is perfectly free from aberrations under the conditions for which it was designed, such as complete freedom in the image of an extended object at infinity. Lenses may be made to this formula of different sizes as regards both aperture and focal length. In the concrete systems the curvatures of the surfaces will vary somewhat from those put down on paper, for the maker, apart from unconscious variations due to inaccuracies of measurement, will deliberately depart from the specification in order to use the tools he has in stock. Consider a particular lens of focal length f in which all the surfaces have their theoretical values with one exception, which is of radius $r + \Delta r$ instead of r, where Δr is small. The defects produced in the image may be represented by θf, where θ is of zero dimensions but is proportional to Δr. From the consideration of systems of different dimensions it is easily seen that θ is of the form

$$\phi \cdot \frac{\Delta r}{r}$$

where ϕ depends only upon the formula on which the lens is constructed and upon the relative aperture and field of view given by the lens. Now the total error is to be kept under a definite limit of the dimensions of a length, which is to be the same for all systems irrespective of their size and aperture. If this limit is denoted by ϵ the permissible variation in r is given by the equation

$$\phi f \frac{\Delta r}{r} = \epsilon \qquad \ldots\ldots(3).$$

In practice the aberrations which cause trouble owing to small variations of the curvatures are those which depend only on the relative aperture. It will be assumed that over the range which determines the distribution law for the tools the first order aberrations introduced by a change of radius are of most importance. Whether this would be the case in practice depends upon the particular design of the lens. It may however be noted that with first order spherical aberration the rate at which the intensity of the light varies on going from the ideal image point is slower than with higher order aberrations, and the defect is therefore relatively more noticeable.

The first defect to be noticed is the departure from achromatism resulting from the change of radius. The longitudinal variation in the position of the focus will be proportional to $f \cdot \frac{\Delta r}{r}$. The latitudinal effect is obtained by multiplying this by the relative aperture x/f. The chromatic condition will thus take the form

$$x \cdot \frac{\Delta r}{r} \leqslant \epsilon_1 \qquad \ldots\ldots(4),$$

where ϵ_1 is some quantity involving the dispersive constants of the glass. The most stringent conditions will be found by giving r/x the minimum value permitted by equation (2). This condition will prove of greatest importance in fixing the spacing of the shallower tools.

For most of the tools the limits determined by the spherical aberration will be of more importance. If the limits to be imposed depend upon the diameter of the circular patch in which the rays meet the paraxial image plane, the limiting condition will be

$$\gamma f \left(\frac{x}{f}\right)^3 \frac{\Delta r}{r} = \epsilon \qquad \ldots\ldots(5),$$

where γ is a spherical aberration coefficient in ϕ and ϵ is the maximum permissible diameter of the patch. If the position of the least circle of confusion be adopted for the image plane the diameter of the image patch will be $\epsilon/4$. An alternative limit will be given by fixing the maximum permissible difference in the length of the path for paraxial and marginal rays between object and image points. If this difference of path limit is denoted by η the corresponding condition is

$$\gamma f \left(\frac{x}{f}\right)^4 \frac{\Delta r}{r} = 16\eta \qquad \ldots\ldots(6).$$

Now variations may take place in any surface, and in every case a different coefficient γ will be found. As a rule the greater the curvature of the surface the

larger the corresponding γ will be. Such conditions as (5) and (6) should therefore be interpreted as applicable to the greatest curvature available for use with the aperture x, the relation between the two being given by equation (2). The equations which have been found suffice to determine a solution for the general problem.

Instead of retaining x as the independent variable it is simpler to introduce ξ defined by

$$\frac{x}{c} = \frac{1-\xi}{\xi} \qquad \ldots.(7),$$

c being a constant. The new variable ξ changes from 0 to 1 as x varies from ∞ to 0. When the focal length of a system becomes very small, the aperture and the focal length tend to equality as a limit. Equation (1) may therefore be rewritten in the form

$$\frac{x}{f} = \xi \qquad \ldots.(8).$$

In very small objectives the aperture tends to equality with the diameter of the most curved surface. The equation for the smallest radius used with the aperture x will thus take the form

$$\frac{r}{c} = \frac{1-\xi}{2\xi} \cdot \frac{\xi+\alpha}{\xi+\beta} \cdot \frac{1+\beta}{1+\alpha} \qquad \ldots.(9),$$

where α and β are numbers fixed to fit the probable requirements. From (9) by differentiation

$$\frac{dr}{r} = -\left\{\frac{1}{\xi(1-\xi)} - \frac{1}{\xi+\alpha} + \frac{1}{\xi+\beta}\right\} d\xi \qquad \ldots.(10),$$

and therefore from (5), (7), (8) and (10)

$$-\frac{\epsilon}{c\gamma}\int_{\infty}^{r}\frac{dr}{\Delta r} = \int_{0}^{\xi}\left\{1-\alpha+\beta+\frac{\alpha(1+\alpha)}{\xi+\alpha} - \frac{\beta(1+\beta)}{\xi+\beta}\right\} d\xi$$

$$= \xi(1-\alpha+\beta) + \alpha(1+\alpha)\log_e\left(1+\frac{\xi}{\alpha}\right) - \beta(1+\beta)\log_e\left(1+\frac{\xi}{\beta}\right) \quad (11).$$

Now if the condition determined by the spherical aberration limit ϵ is a satisfactory criterion for the determination of the curvatures of the entire series of tools, the serial number n of any tool reckoned from zero as the number of a plane will be given by

$$\int_{\infty}^{r}\frac{dr}{\Delta r} = 2n \qquad \ldots.(12).$$

But since the right side of (11) remains finite as ξ varies between 0 and 1, it follows that the number of different curvatures required for all purposes is a finite number N. The unknown quantities on the left of (11) can thus be eliminated, yielding the equation

$$\frac{n}{N} = \frac{\xi(1-\alpha+\beta) + \alpha(1+\alpha)\log_e\left(1+\frac{\xi}{\alpha}\right) - \beta(1+\beta)\log_e\left(1+\frac{\xi}{\beta}\right)}{1-\alpha+\beta+\alpha(1+\alpha)\log_e\left(1+\frac{1}{\alpha}\right) - \beta(1+\beta)\log_e\left(1+\frac{1}{\beta}\right)} \quad (13)$$

for finding the value of ξ associated with the nth curvature out of a complete set of N different curvatures. The radius is determined by substituting this value of ξ in equation (9).

For the earlier members of the series ξ will be small, and the numerator on the right of equation (13) reduces to ξ. It follows from the proportionality of ξ and n in these cases that the low numbers of the series have their curvatures in arithmetical progression.

From equation (5) the density with which the tools are distributed, which may be measured by the difference between the ratio of the curvatures of successive tools and unity, is proportional to

$$f \cdot \left(\frac{x}{f}\right)^3$$

or to

$$\xi\,(1-\xi)$$

and thus reaches a maximum when $\xi = \frac{1}{2}$, which is in the neighbourhood of the radius

$$\frac{c}{2} \cdot \frac{1+2\alpha}{1+2\beta} \cdot \frac{1+\beta}{1+\alpha} \qquad \ldots.(14).$$

The proportion of tools on the shallow side of this maximum position is given by writing $\frac{1}{2}$ for ξ on the right of equation (13).

If the proper criterion for the spacing of the tools is assumed to depend upon the maximum error of phase caused by a small error in a tool, equation (6) must be used instead of equation (5). The result obtained is

$$\frac{n}{N} = \frac{\frac{1}{2}\xi^2(1-\alpha+\beta)+\xi(\alpha-\beta)(1+\alpha+\beta)-\alpha^2(1+\alpha)\log_e\left(1+\frac{\xi}{\alpha}\right)+\beta^2(1+\beta)\log_e\left(1+\frac{\xi}{\beta}\right)}{\frac{1}{2}(1-\alpha+\beta)+(\alpha-\beta)(1+\alpha+\beta)-\alpha^2(1+\alpha)\log_e\left(1+\frac{1}{\alpha}\right)+\beta^2(1+\beta)\log_e\left(1+\frac{1}{\beta}\right)} \qquad \ldots.(15).$$

The tools are distributed with a density proportional to

$$\xi^2\,(1-\xi)$$

and the maximum is reached when $\xi = \frac{2}{3}$ in the neighbourhood of the radius

$$\frac{c}{4} \cdot \frac{2+3\alpha}{2+3\beta} \cdot \frac{1+\beta}{1+\alpha} \qquad \ldots.(16).$$

When ξ is small the numerator on the right of (15) reduces to $\frac{1}{3}\xi^2$, the curvatures of the low numbered tools being proportional to the square root of their number.

It is necessary to decide which formula is the more correct to adopt as a guide. An appeal to a comparison with manufacturers' lists would probably favour formula (13) rather than (15), but this would be to prejudge the question since lenses have been designed on calculations which adopted the former measurement as the criterion rather than the latter. On the other hand there appears to be ground for thinking that there is greater room for improvement in short focus systems of wide aperture than in larger instruments such as telescopes, and such a change would correspond to the substitution of the latter condition for the former. It has however to be remembered that the relatively greater perfection reached in telescopes is brought about by departures from the spherical form in the refracting surfaces, and it may well prove to be the case that owing to the difficulty of reconciling the various conditions in such instruments as photographic lenses the earlier condition is the more reasonable one to apply when spherical surfaces alone are

considered, and that the attainment of the second condition will in general require departures from the spherical form. If this view is correct the spacing of spherical tools must undoubtedly be determined according to equation (13). The suggested standard set of tools appended to this paper has been calculated on the supposition that this contention is true.

It may very well be objected that the consideration of first order spherical aberration alone will lead to invalid results, particularly in small objectives where the ratio of the shortest radius to the aperture is approaching $\frac{1}{4}$. It has on the other hand to be remembered that in systems of this kind these abnormally steep surfaces are so placed that they refract light practically without aberration. A general calculation cannot be made as the steepest surfaces are usually internal and the effect of a change in such surfaces depends upon the errors present in the portions of the system on either side of the surface concerned. It appears however desirable to investigate the case which can be dealt with without any assumptions, that in which the nominal system is perfectly corrected and the surface to be varied is external. It will be sufficient to assume that this is the final surface of the system. The alteration of this surface is equivalent to adding on to the corrected system a very deep meniscus lens of radii r and $r + \Delta r$. Suppose that the rays from the axial point of the object for which the system is corrected are brought to a focus at a distance $(1 - m)\, r$ from the last surface of the system. It is easy to show that the addition of the thin meniscus lens will increase this distance for paraxial rays by

$$-(1 - m)^2 (\mu - 1)\, \Delta r.$$

Let ϕ and ψ be the angles made in air by a ray with the normal to the last surface of the system and with the axis respectively, and let the angles in glass be denoted by accented letters. Then

$$\sin \phi = \mu \sin \phi' = m \sin \psi,$$

and

$$\psi' = \phi' - \phi + \psi.$$

The usual formula for the first order aberration of a thin lens shows that the longitudinal aberration introduced by the additional lens is

$$\tfrac{1}{2} m (1 - m)^2 \{2\mu + m(\mu + 2)\} \frac{\mu - 1}{\mu} \Delta r \tan^2 \psi \qquad \ldots\ldots(17),$$

and it is not difficult to show that the exact longitudinal displacement is

$$\left\{1 - \frac{\mu \sin \psi'}{\sin \psi} - m \cos \psi \left(\frac{1}{\cos \phi} - \frac{1}{\mu \cos \phi'}\right)\left(m - \frac{\mu \sin \psi'}{\sin \psi}\right)\right\} \Delta r \quad \ldots(18).$$

For the comparison of the exact aberration with the first order value it is desirable to simplify (18) in a form free from ψ', thus enabling all expressions involving ϕ and ϕ' only to be separated from those involving ψ. This may be done by noting that

$$1 - \frac{\mu \sin \psi'}{\sin \psi} = (1 - m)(\cos \phi - \mu \cos \phi') \frac{\cos \phi + \cos \psi}{1 + \cos(\phi + \psi)},$$

the substitution of which in (18) reduces that expression to

$$-(1 - m)^2 \left(\frac{1}{\cos \phi} - \frac{1}{\mu \cos \phi'}\right) \left\{\mu \cos \phi' + m \sin^2 \psi \cdot \frac{m \cos \psi + 2\mu \cos \phi'}{1 + \cos(\phi + \psi)}\right\} \Delta r$$

$$\text{or} \quad -(1-m)^2(\mu-1)\Delta r - m(1-m)^2\left\{\frac{m(\mu^2-1)}{\mu\cos\phi(\cos\phi+\cos\phi')} + \left(\frac{1}{\cos\phi}-\frac{1}{\mu\cos\phi'}\right)\frac{m\cos\psi+2\mu\cos\phi'}{1+\cos(\phi+\psi)}\right\}\Delta r\sin^2\psi \quad ..(19).$$

The calculation of the exact aberrations is not likely to be simplified by taking out common factors, such as by the use of the relation $\mu^2-1=\mu^2\cos^2\phi'-\cos^2\phi$. The ratio of the exact latitudinal aberration to that given by the first order expression is equal to the ratio of the corresponding longitudinal expressions, which by (17) and (19) is

$$\frac{1+\cos 2\psi}{2\mu+m(\mu+2)}\left\{\frac{m(\mu+1)}{\cos\phi(\cos\phi+\cos\phi')} + \frac{\mu}{\mu-1}\left(\frac{1}{\cos\phi}-\frac{1}{\mu\cos\phi'}\right)\frac{m\cos\psi+2\mu\cos\phi'}{1+\cos(\phi+\psi)}\right\} \quad ....(20).$$

The following table shows the values of this ratio as ϕ changes from 60° to − 60° by steps of 10°, and ψ from 30° to − 30° in steps of 5°. A change in sign of both ϕ and ψ does not alter the value of the ratio. The refractive index has been given the mean value 1·55. It may be pointed out that in photographic lenses the usual limits for ϕ and ψ are approximately 30° and 5°, and that cases in which the latter quantity exceeds 8° are rare. The zone at which the ray is refracted subtends an angle $2(\phi-\psi)$ at the centre of the surface.

These numerical results fully justify, when the uncertainty of some other assumptions is considered, the adoption of the first order spherical aberration alone as the basis on which a series of tools may be calculated. It is of interest to note in passing that the table, as well as the formula from which it is calculated, suggests that a superior limit to the aberration of very strongly curved meniscus lenses may be found by multiplying the values given by the first order aberration formula by $\sec^2\phi$.

Table I.

Values of ϕ.

Values of ψ.	0°	10°	20°	30°	40°	50°	60°
− 30°	·804	·823	·942	1·084	1·135	1·541	2·297
− 25°	·862	·885	1·123	1·013	1·277	1·719	2·562
− 20°	·910	·938	1·052	1·121	1·394	1·856	2·801
− 15°	·949	·986	1·031	1·199	1·489	2·008	3·012
− 10°	·977	1·015	1·082	1·256	1·564	2·118	3·193
− 5°	·994	1·018	1·113	1·295	1·619	2·204	3·341
0°	1·000	1·030	1·128	1·318	1·655	2·264	3·455
5°	·994	1·027	1·129	1·323	1·671	2·300	3·533
10°	·977	1·013	1·117	1·315	1·668	2·309	3·574
15°	·949	·986	1·091	1·291	1·646	2·292	3·577
20°	·910	·948	1·053	1·251	1·604	2·224	3·541
25°	·862	·900	1·002	1·196	1·543	2·181	3·464
30°	·804	·841	·941	1·128	1·464	2·086	3·352

The slight irregularities which appear in the table when $\phi+\psi$ is small are due to the fact that both numerator and denominator are small, the higher order aberrations vanishing for an object point near to, but not quite coincident with,

that for which the first order aberration is zero. In this neighbourhood the ratio must change sign twice, once through zero and once through infinity. The range within which this takes place is evidently very small, and the errors made by assuming that the aberrations of all orders vanish with those of the first order will not be serious.

Before the expressions which have been found can be applied practically it is necessary to determine values for the constants c, α, and β. From equations (7) and (8) the aperture f/σ can be obtained for focal lengths up to a maximum value of $c\sigma\ (\sigma - 1)$. Corresponding values are

σ	2	3	4	5	6	7	8	9	10
(f/c) max.	2	6	12	20	30	42	56	72	90

The comparison of these figures with those of various lists of photographic lenses suggests a value of c between 0·5 and 1·0 if measurements are made in inches.

The limit which the ratio r/x approaches as x becomes large may not unreasonably be fixed as 1·5. This gives

$$\alpha = 3\beta/(1 - 2\beta).$$

To determine the remaining constant the value of the aperture at which the limiting value of r/x is unity may be considered. If this aperture be denoted by z

$$1 + 1/\beta = z/c.$$

Some check on the correctness of the value chosen for z should be obtainable from existing lists of tools by finding the value of r at which the ratio of the curvatures of successive tools is nearest to unity. Let this value of the radius be denoted by $\bar{r}$. The substitution of the above values for α and β in equation (14) gives

$$\frac{\bar{r}}{c} = \frac{1}{2} \cdot \frac{z + 3c}{z + c}$$

with corresponding values

z/c	1	2	3	4	5	6	7	8
$\bar{r}/c$	1	·833	·75	·7	·667	·643	·625	·611

showing that considerable uncertainty in the value of z involves only a small change in $\bar{r}$. The dependence of r/x upon the choice of z is given by

$$\frac{r}{x} = \frac{1}{2} \cdot \frac{z + 3x}{z + x}.$$

In the numerical application of the above results it will be assumed that when the inch is the unit of measurement $c = \frac{3}{8}$, $\alpha = 1$, $\beta = \frac{1}{8}$ corresponding to $z = 4$ and $\bar{r} = \frac{3}{4}$. The formula for the calculation now takes the form

$$\frac{n}{N} = \frac{\cdot 1\xi + \log_e (1 + \xi) - \cdot 12 \log_e (1 + 5\xi)}{\cdot 1 + \log_e 2 - \cdot 12 \log_e 6}$$

$$= \frac{\cdot 04343\xi + \log_{10} (1 + \xi) - \cdot 12 \log_{10} (1 + 5\xi)}{\cdot 25108} \quad \ldots\ldots(21).$$

Instead of calculating the radius in inches it is preferable to take 1 mm. as the unit. The constant c has therefore to be multiplied by 25, and the formula for the radius becomes

$$r = \frac{5\,(1 - \xi^2)}{\xi\,(\cdot 2 + \xi)} \quad \ldots\ldots(22).$$

As the denominator in (21) is approximately ·25, the numerator can be multiplied by $4N$ to give the values of ξ corresponding to a list of approximately N curvatures.

Table II shows approximate values for every tenth ξ calculated in this way on the supposition that N is one thousand. It will be seen that the resulting list agrees in its general character with that which would be given by the simpler formula $\xi = n/N$. The substitution of the latter would introduce three more tools between numbers 0 and 50, and take away five between 50 and 150; in the first five hundred there would be twenty-nine tools to be deleted, and so on. The general tendency of such a change would be to increase the number of tools of short radius to some extent at the expense of those of longer radius. The alteration necessary in the original assumptions to yield this simpler formula without altering equation (9) is the substitution for (8) of the more complex relation

$$\frac{x}{f} = \xi \left\{ 1 + \frac{\xi\,(1-\xi)\,(\alpha-\beta)}{(\xi+\alpha)\,(\xi+\beta)} \right\}^{-\frac{1}{2}} \qquad \ldots\ldots(23).$$

The consequent changes in the greatest permissible focal lengths for given relative apertures are shown in the accompanying table:

Relative aperture	*f*/1	*f*/2	*f*/3	*f*/4	*f*/5	*f*/6	*f*/7	*f*/8	*f*/9	*f*/10
Greatest *f*/*c* by (8)	0	2	6	12	20	30	42	56	72	90
Greatest *f*/*c* by (23)	0	1·79	5·09	10·24	17·20	26·01	36·69	49·28	63·79	80·22

It may be noted that the second line of values of f/c may be altered in a common ratio to bring its values for f approximately into agreement with those of the first line by making a change in the value of c. As the figures stand the two series agree for very short and for very great focal lengths. An increase in c of about 12 per cent. would prove a suitable change for moderate focal lengths.

The general features of the way in which the tools are distributed may be shown diagrammatically. To obtain a compact diagram it is desirable to take a logarithmic scale for the radius, which is the independent variable and is measured horizontally. The height is proportional to the number of tools between limits corresponding to a constant length of the base. If the tools were arranged according to a geometrical progression the resulting curve would be a horizontal straight line. For in such a case there would be the same number of tools between 1 and 2 mm. as between 10 and 20 mm., or 100 and 200 mm., or 1000 and 2000 mm. Each of these intervals is represented on a logarithmic scale by an equal length, and the corresponding parts of the distribution curve must therefore be at the same distance from the base. If the number of tools in the progression were doubled by halving the interval, the straight line would be displaced to double its original distance from the base. The actual curves shown in Fig. 1 differ very appreciably from a straight line, showing that the spacing reached departs sensibly from a geometrical progression. Curve A is the one adopted in this investigation as the basis for a standard tool list. Curve B is the corresponding distribution given by the simpler law $\xi = n/N$. Curve C would have to be substituted for A if equation (6), corresponding to a difference of phase condition, were substituted for equation (5) in the determination of the list.

If any manufacturer's list were examined and plotted in a similar way a very irregular curve would be found because it will inevitably happen that no call has ever arisen for certain tools of a complete system. To obtain a comparison it is convenient to plot the number of tools existing in a finite interval, the resulting chart presenting a series of steps. On the assumption that in a perfectly complete series about 1000 different curvatures will be required, and that steps corresponding

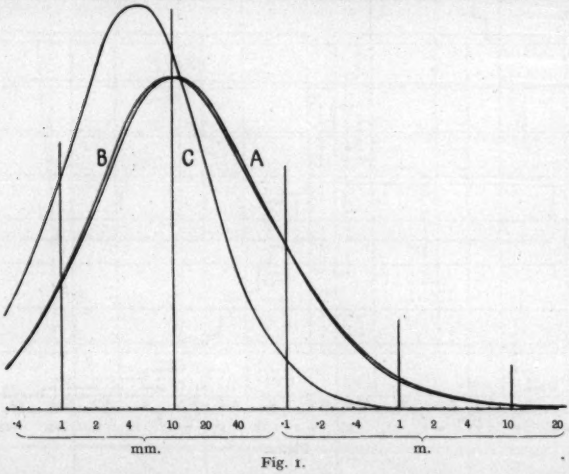

Fig. 1.

to a logarithmic difference of 0·1 are adopted, curve *A* of Fig. 1 becomes the full line stepped curve of Fig. 2. Two of our best known manufacturers have courteously permitted curves to be added showing the actual distribution of the tools they use. The curve denoted by crosses is a tool list of some five years ago. It will be observed that in its general shape it approximates closely to the curve derived from theory. An inspection of the list shows that there are many gaps, and that if these were filled on a basis corresponding to the smallest interval for which any decided need is indicated in the list, the resulting curve would be raised in height approximately to the level of the full theoretical curve. The curve shown by circles is the list of tools above 1 cm. radius of a maker who considers that it is about sufficient to meet any demand that is likely to be made from it. The agreement between this curve and the theoretical one in the region in which any correspondence may be expected is extraordinary close, and suggests that the value of one thousand provisionally adopted for the total number of curvatures is about correct. As agreement upon this question is of the utmost importance it is very desirable that manufacturers should say whether their experience shows that this value of *N*

results in suitable intervals between successive tools. For this purpose lists have been prepared giving the radii of the tools in millimetres and inches. A third table gives the curvatures in dioptres. The variation of the curvature interval ΔR with

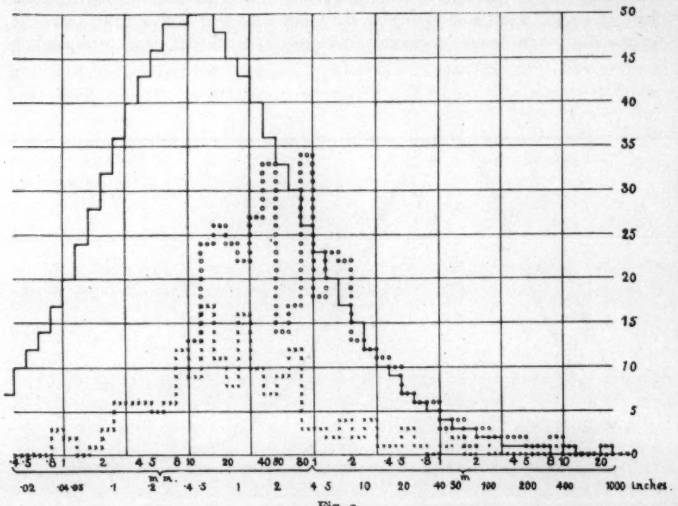

Fig. 2.

the logarithm of the radius is shown in Fig. 3. From this and from the table of curvatures it is easily seen that the chromatic condition in equation (4) is properly met, and that the tools are everywhere close enough to enable any desired focal length to be attained with reasonable accuracy.

It may be contended that in spite of its satisfactory general features the proposed list suffers under a disadvantage in comparison with one based on a geometrical progression in that it does not offer series of tools bearing the same relation to one another and therefore suitable for the manufacture of similar lenses of different focal lengths. This objection will not however withstand examination, for the list has been prepared on a basis which automatically secures such an interval between successive tools that, under the most trying conditions, a satisfactory result will always be obtained by selecting the nearest tool. This liberty will naturally be used with discrimination when variations occur with a number of the radii, and as far as possible departures from the desired curvatures will all be made in the same direction, thus cutting out aberrations at the expense of a slight difference of focal length. The general convenience of the list may however be best demonstrated by taking

at random a searching test case. Suppose that a well corrected lens of five inches focal length has been designed and that the tools used are numbers 50, 100, 150, 200, 250, 300, 350 and 400. It is desired to make similar lenses of focal lengths 6, 7, 8, 10, 12, 15 and 20 inches, with an accuracy of focal length varying from about 1 per cent. for the shorter focal lengths to about 2 per cent. for the longer ones. It will be realised that the difficulties involved in moving from the shorter focal length

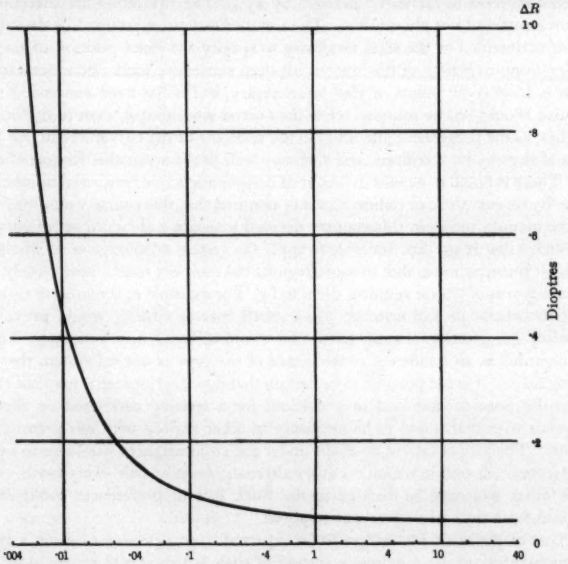

Fig. 3.

to the longer ones are greater than if the process were reversed, as the intervals are greater the greater the radius of the tool. Moreover the difficulty of obtaining a good fit increases rapidly with the number of surfaces involved. Possible lenses in accordance with these conditions are given below; the focal lengths are stated as accurately as they can be estimated. The number of the tool is given first and is followed by the number of parts in a thousand by which its radius differs from the value which would secure exact similarity to the original lens. As all the lenses will be of approximately equal diameter considerable errors in the tools of small number may be of less consequence than small errors in those of large number. The figures show conclusively that no weight whatever can be attached to the

supposed superiority of a geometrically progressive series of tools. On the other hand such a series is evidently quite wanting in the elasticity which is present in the series proposed.

In the accompanying tables of radii and curvatures the calculations have been carried out for every one of the first seven hundred curvatures, and thereafter every tenth curvature has been given. The radii of the first five hundred of these have been converted to inches by division by 25·4. The two tables are therefore exact equivalents and not alternatives. The number of figures retained in the tables has been maintained at the level necessary to specify the exact position of the tool to at least one-twentieth of the interval between successive tools. So exact a specification is from some points of view unnecessary, but it has been concluded that the figures should not be rounded off to the nearest simple equivalent in millimetres or inches, as the progressive alteration in the relations of the curvatures to one another would thereby be sacrificed, and this may well prove a valuable feature of the list.

There is much to be said in favour of designating a tool by a serial number rather than by its curvature or radius, and it is assumed that this course would be followed if the manufacturers in this country decided to adopt a standard set of curvatures. In such a case it appears desirable to apply the system adopted over its whole range, though this may mean that in some regions the tools are much more closely packed than any manufacturer requires them to be. For example at the present time in the neighbourhood of tool number 500 a much coarser spacing would prove ample, possibly the selection of every tenth tool would suffice in any workshop. This is to be regarded as an accidental consequence of the type of optical system that is now being made. It is not possible to be certain that new developments may not radically alter the position and lead to a demand for a spacing correlated on theoretical grounds with that found to be necessary in other regions with our present instruments. The proper course to adopt under the circumstances would be to express a preference in a certain region for every alternate, every fourth, every tenth, or whatever other tool may be decided upon. Such lists of preferences could easily be revised from time to time as was required.

One of the main advantages to be expected from agreement among a group of manufacturers to use a common system of tools is that it will render possible the specialised manufacture of both tools and proof-plates. A firm undertaking such work would be justified in installing special machinery and measuring equipment on a scale that is unattainable by each individual manufacturer. It should then be possible to procure these articles not only at a lower cost than at present but with increased accuracy. There is no reason why this manufacturer should not be able to fulfil orders from stock provided agreement were reached on other dimensions than those discussed in this paper. It is in the hope of assisting in the attainment of such an end and thereby benefiting the optical industry of this country that this investigation has been carried out.

It may not be without interest in conclusion to say a word on the method adopted in making the calculations. The solution of an equation like (21) for integral values of n cannot be carried out directly. A very rough approximation to ξ was obtained

Tool No.	Error
$f = 5{\cdot}96$	
365	− 1
317	0
270	− 1
223	+ 2
177	+ 1
132	− 3
87	− 2
43	− 4
$f = 6{\cdot}93$	
336	− 1
290	+ 1
246	− 2
202	0
159	+ 1
117	+ 5
77	− 3
37	+ 13
$f = 7{\cdot}93$	
311	0
268	− 3
225	+ 1
184	+ 1
144	+ 3
106	− 3
69	− 6
33	+ 5
$f = 9{\cdot}99$	
271	0
231	0
193	− 1
156	+ 3
121	+ 4
88	0
56	+ 8
27	− 8
$f = 12{\cdot}05$	
241	− 1
204	+ 1
169	+ 2
136	+ 2
105	− 1
75	+ 7
47	− 2
22	+ 25
$f = 19{\cdot}57$	
174	− 1
145	0
118	+ 3
94	− 5
71	− 3
50	− 3
31	+ 1
14	+ 18

Tool No.	Error
$f = 5{\cdot}99$	
364	− 1
316	+ 1
269	0
222	+ 3
176	+ 4
131	+ 2
86	+ 7
43	− 9
$f = 6{\cdot}97$	
335	− 1
289	+ 1
245	− 2
201	+ 1
158	+ 4
117	− 1
76	+ 6
37	+ 7
$f = 8{\cdot}01$	
309	+ 1
266	− 1
224	− 2
183	− 1
143	+ 2
105	− 1
68	+ 1
33	− 5
$f = 10{\cdot}12$	
269	− 1
229	+ 1
191	+ 1
155	− 1
120	+ 2
87	+ 1
56	− 5
27	− 20
$f = 14{\cdot}78$	
211	− 1
177	+ 3
146	0
117	− 3
89	+ 3
64	− 7
40	0
19	− 21
$f = 20{\cdot}00$	
171	+ 1
143	− 3
116	+ 3
92	− 1
70	− 8
49	0
30	+ 15
14	− 3

Tool No.	Error
$f = 6{\cdot}02$	
363	0
315	+ 1
268	+ 1
222	− 1
176	0
131	− 3
86	+ 2
42	+ 12
$f = 7{\cdot}03$	
333	+ 1
288	− 1
243	+ 2
200	0
158	− 4
116	+ 2
76	− 2
37	− 1
$f = 8{\cdot}10$	
307	0
264	0
222	0
181	+ 3
142	0
104	0
67	+ 7
33	− 16
$f = 11{\cdot}88$	
243	0
206	0
171	0
138	− 3
106	+ 1
76	+ 5
49	− 11
23	− 8
$f = 14{\cdot}94$	
209	+ 2
176	0
145	− 1
116	− 3
88	+ 5
63	0
40	− 11
18	+ 24
$f = 20{\cdot}53$	
168	0
140	− 1
114	0
90	0
68	0
48	− 3
30	− 11
14	− 30

Tool No.	Error
$f = 6{\cdot}07$	
361	+ 1
314	− 1
267	− 1
221	+ 4
175	0
130	− 1
86	− 6
42	+ 4
$f = 7{\cdot}08$	
332	0
287	− 2
242	+ 2
199	+ 1
157	− 2
116	− 5
75	+ 7
37	− 8
$f = 9{\cdot}92$	
272	+ 1
232	+ 1
194	− 1
157	+ 1
122	0
89	− 7
57	− 5
27	0
$f = 11{\cdot}97$	
242	0
205	0
170	0
137	− 1
106	− 6
76	− 2
48	+ 5
23	− 15
$f = 15{\cdot}19$	
207	− 1
174	− 1
143	+ 1
114	+ 2
87	+ 3
62	+ 2
39	0
18	+ 7

by assuming it to be n/N. This was done for every hundredth n, and the value found for the right side of the equation suggested a correction which enabled the next approximation to be fairly close to the desired value. By interpolation the exact value to about seven places of decimals was found in about two further trials. Between each of these values of ξ nine others were interpolated, using the first differential coefficient of (21) as a guide. These values were corrected to the same order of accuracy as the others in one trial on the average.

The next step was to calculate from formula (22) the *curvature* of every tenth tool for the smaller values of n and the *radius* for the larger values. These were of course selected as they were varying more uniformly than their reciprocals. The first, second, third,... orders of differences were formed from these tenth values, and nine means interpolated between them by finding the differences corresponding to the increased number of terms. Let the original series be $A_1, A_2, A_3, \ldots$, and the leading members of the first, second, third, ... orders of differences $a_1, a_2, a_3, \ldots$. Then it is easy to see that

$$A_{n+1} = A_1 + {}^nC_1 a_1 + {}^nC_2 a_2 + {}^nC_3 a_3 + \ldots$$

which may be written symbolically as

$$A_{n+1} = (1 + a)^n$$

where

$$A_1 = a_0.$$

If $p - 1$ means are inserted between each term, and the new series is denoted by the substitution of B's for A's, the $(n + 1)$th term of the A series corresponds to the $(np + 1)$th term of the B series, so that symbolically

$$(1 + a)^n = (1 + b)^{np}$$

for all values of n.

For n zero

$$B_1 = A_1.$$

Also

$$b_1 = (1 + a)^{1/p} - 1$$

where the right side is to be expanded, and generally

$$b_q = \{(1 + a)^{1/p} - 1\}^q.$$

In the present application $p = 10$.

This method proved very convenient, and the resulting series run very smoothly. A check on the accuracy obtained was made by direct calculation from the original formula for a number of the curvatures derived solely by interpolation. In particular nearly all the first hundred tools were calculated by the direct method.

The author is under great obligations to Miss Gillman who has rendered very valuable assistance in all the numerical work involved. Whatever value the tabulated curvatures and radii may have is very largely due to the great care she has taken in their preparation.

Note.

In the following tables the leading digits of the serial number n of the tool are placed in the first column, and subsequent digits at the heads of the remaining columns, the quantity given by the table for any particular value of n occurring where the row containing the leading digits crosses the column with the concluding figures. Thus from Table III the radius corresponding to $n = 243$ is 46·96 mm., the equivalent in British units derived from Table V being 1·8489 inches.

Table II.

Approximate values of $n - 1000\xi$ for every tenth n.

	00	10	20	30	40	50	60	70	80	90
0	0	− 1·27	− 2·14	− 2·69	− 2·99	− 3·08	− 3·01	− 2·79	− 2·45	− 2·01
1	− 1·49	− 0·90	− 0·24	0·47	1·22	2·01	2·84	3·68	4·55	5·44
2	6·34	7·25	8·17	9·08	10·00	10·92	11·84	12·75	13·65	14·54
3	15·42	16·29	17·14	17·98	18·80	19·61	20·40	21·16	21·91	22·63
4	23·33	24·01	24·66	25·29	25·90	26·47	27·02	27·55	28·04	28·51
5	28·94	29·35	29·72	30·07	30·38	30·66	30·91	31·13	31·31	31·46
6	31·58	31·66	31·71	31·72	31·70	31·64	31·55	31·42	31·25	31·04
7	30·80	30·52	30·21	29·85	29·46	29·02	28·55	28·04	27·49	26·90
8	26·27	25·61	24·90	24·15	23·35	22·52	21·65	20·73	19·78	18·78
9	17·74	16·66	15·53	14·36	13·15	11·90	10·60	9·26	7·87	6·45
10	4·98	—	—	—	—	—	—	—	—	—

Table III.

PROPOSED STANDARD GLASS-WORKING TOOLS.

Radii in Millimetres.

	0	1	2	3	4	5	6	7	8	9
0	∞	21600	10800	7160	5350	4270	3550	3030	2640	2340
1	2099	1902	1738	1599	1480	1377	1287	1207	1136	1073
2	1016	965	918	875	836	800	767	736	708	681
3	656·6	633·5	611·8	591·5	572·3	554·3	537·3	521·2	505·9	491·5
4	477·8	464·7	452·3	440·5	429·2	418·4	408·1	398·3	388·9	379·8
5	371·1	362·8	354·8	347·1	339·7	332·6	325·7	319·1	312·7	306·5
6	300·5	294·8	289·2	283·8	278·6	273·6	268·6	263·9	259·3	254·8
7	250·5	246·3	242·1	238·2	234·3	230·6	226·9	223·4	219·9	216·5
8	213·2	210·0	206·9	203·9	200·9	198·0	195·2	192·4	189·7	187·1
9	184·5	182·0	179·5	177·1	174·8	172·5	170·2	168·0	165·9	163·8
10	161·72	159·70	157·72	155·78	153·88	152·02	150·19	148·40	146·65	144·92
11	143·24	141·58	139·95	138·36	136·79	135·25	133·74	132·26	130·81	129·38
12	127·97	126·59	125·23	123·90	122·59	121·30	120·03	118·79	117·56	116·36
13	115·17	114·01	112·86	111·73	110·62	109·52	108·45	107·39	106·34	105·31
14	104·30	103·30	102·32	101·35	100·40	99·46	98·54	97·62	96·73	95·84
15	94·97	94·10	93·26	92·42	91·59	90·78	89·98	89·18	88·40	87·63
16	86·87	86·12	85·38	84·65	83·93	83·22	82·52	81·83	81·14	80·47
17	79·80	79·14	78·49	77·85	77·21	76·59	75·97	75·36	74·76	74·16
18	73·57	72·99	72·41	71·84	71·28	70·73	70·18	69·64	69·10	68·57
19	68·05	67·53	67·02	66·51	66·01	65·52	65·03	64·55	64·07	63·59
20	63·13	62·66	62·21	61·75	61·30	60·86	60·42	59·99	59·56	59·13
21	58·71	58·30	57·89	57·48	57·08	56·68	56·28	55·89	55·50	55·12
22	54·74	54·37	53·99	53·63	53·26	52·90	52·54	52·19	51·84	51·49
23	51·15	50·81	50·47	50·14	49·80	49·48	49·15	48·83	48·51	48·20
24	47·88	47·57	47·27	46·96	46·66	46·36	46·07	45·77	45·48	45·19
25	44·91	44·63	44·35	44·07	43·79	43·52	43·25	42·98	42·71	42·45
26	42·19	41·93	41·67	41·42	41·16	40·91	40·67	40·42	40·18	39·93
27	39·69	39·45	39·22	38·98	38·75	38·52	38·29	38·07	37·84	37·62
28	37·40	37·18	36·96	36·74	36·53	36·32	36·11	35·90	35·69	35·48
29	35·28	35·08	34·88	34·68	34·48	34·28	34·09	33·89	33·70	33·51
30	33·321	33·134	32·947	32·762	32·579	32·397	32·216	32·037	31·858	31·682
31	31·506	31·332	31·159	30·987	30·817	30·648	30·480	30·313	30·147	29·983
32	29·820	29·658	29·497	29·337	29·179	29·021	28·865	28·710	28·555	28·402
33	28·250	28·099	27·949	27·800	27·652	27·506	27·360	27·215	27·071	26·928
34	26·786	26·645	26·505	26·366	26·228	26·091	25·955	25·819	25·685	25·551
35	25·418	25·286	25·155	25·025	24·896	24·768	24·640	24·513	24·387	24·262
36	24·138	24·014	23·892	23·770	23·649	23·528	23·409	23·290	23·172	23·054
37	22·938	22·822	22·707	22·592	22·478	22·365	22·253	22·142	22·031	21·920
38	21·811	21·702	21·594	21·486	21·379	21·273	21·167	21·062	20·958	20·854
39	20·751	20·649	20·547	20·445	20·345	20·245	20·145	20·046	19·948	19·850
40	19·753	19·657	19·561	19·465	19·370	19·276	19·182	19·089	18·996	18·904

Table III continued on p. 256.

Table IV.

PROPOSED STANDARD GLASS-WORKING TOOLS.

Curvatures in Dioptres.

	0	1	2	3	4	5	6	7	8	9
0	·000	·046	·093	·140	·187	·234	·282	·330	·379	·427
1	·476	·526	·575	·625	·676	·726	·777	·828	·880	·932
2	·984	1·037	1·089	1·142	1·196	1·250	1·304	1·358	1·413	1·468
3	1·523	1·579	1·634	1·691	1·747	1·804	1·861	1·919	1·977	2·035
4	2·093	2·152	2·211	2·270	2·330	2·390	2·450	2·511	2·572	2·633
5	2·694	2·756	2·818	2·881	2·944	3·007	3·070	3·134	3·198	3·263
6	3·327	3·393	3·458	3·524	3·590	3·656	3·723	3·790	3·857	3·924
7	3·992	4·061	4·129	4·198	4·267	4·337	4·407	4·477	4·548	4·618
8	4·690	4·761	4·833	4·905	4·978	5·051	5·124	5·197	5·271	5·345
9	5·420	5·495	5·570	5·645	5·721	5·798	5·874	5·951	6·028	6·106
10	6·184	6·262	6·340	6·419	6·499	6·578	6·658	6·739	6·819	6·900
11	6·982	7·063	7·145	7·228	7·310	7·393	7·477	7·561	7·645	7·729
12	7·814	7·899	7·985	8·071	8·157	8·244	8·331	8·418	8·506	8·594
13	8·683	8·772	8·861	8·950	9·040	9·131	9·221	9·312	9·404	9·496
14	9·588	9·680	9·773	9·866	9·960	10·054	10·149	10·243	10·339	10·434
15	10·530	10·626	10·723	10·820	10·918	11·016	11·114	11·213	11·312	11·411
16	11·511	11·611	11·712	11·813	11·914	12·016	12·118	12·221	12·324	12·428
17	12·531	12·636	12·740	12·845	12·951	13·057	13·163	13·270	13·377	13·484
18	13·592	13·701	13·810	13·919	14·029	14·139	14·249	14·360	14·471	14·583
19	14·695	14·808	14·921	15·035	15·148	15·263	15·378	15·493	15·609	15·725
20	15·841	15·958	16·076	16·194	16·312	16·431	16·550	16·670	16·790	16·911
21	17·032	17·153	17·275	17·398	17·520	17·644	17·768	17·892	18·017	18·142
22	18·268	18·394	18·521	18·648	18·776	18·904	19·032	19·161	19·291	19·421
23	19·552	19·683	19·814	19·946	20·079	20·212	20·345	20·479	20·614	20·749
24	20·884	21·020	21·157	21·294	21·431	21·569	21·708	21·847	21·987	22·127
25	22·267	22·409	22·550	22·693	22·835	22·979	23·122	23·267	23·412	23·557
26	23·703	23·850	23·997	24·144	24·293	24·441	24·591	24·740	24·891	25·042
27	25·193	25·345	25·498	25·651	25·805	25·959	26·114	26·270	26·426	26·583
28	26·740	26·898	27·056	27·215	27·375	27·535	27·696	27·857	28·019	28·182
29	28·345	28·509	28·673	28·838	29·004	29·170	29·337	29·504	29·673	29·841
30	30·011	30·181	30·351	30·523	30·695	30·867	31·040	31·214	31·389	31·564
31	31·740	31·916	32·093	32·271	32·450	32·629	32·809	32·989	33·170	33·352
32	33·535	33·718	33·902	34·086	34·272	34·458	34·644	34·832	35·020	35·209
33	35·398	35·588	35·779	35·971	36·163	36·356	36·550	36·745	36·940	37·136
34	37·333	37·530	37·728	37·927	38·127	38·328	38·529	38·731	38·934	39·137
35	39·341	39·547	39·753	39·959	40·167	40·375	40·584	40·794	41·005	41·216
36	41·428	41·642	41·855	42·070	42·286	42·502	42·719	42·937	43·156	43·376
37	43·596	43·818	44·040	44·263	44·487	44·712	44·937	45·164	45·391	45·620
38	45·849	46·079	46·310	46·542	46·775	47·008	47·243	47·478	47·715	47·952
39	48·190	48·430	48·670	48·911	49·153	49·396	49·640	49·885	50·130	50·377
40	50·625	50·874	51·123	51·374	51·626	51·878	52·132	52·387	52·642	52·899

Table IV continued on p. 257.

Table III (*continued*).

PROPOSED STANDARD GLASS-WORKING TOOLS.

Radii in Millimetres.

	0	1	2	3	4	5	6	7	8	9
40	19·753	19·657	19·561	19·465	19·370	19·276	19·182	19·089	18·996	18·904
41	18·812	18·721	18·631	18·540	18·451	18·362	18·273	18·185	18·098	18·011
42	17·924	17·838	17·752	17·667	17·583	17·498	17·415	17·331	17·249	17·166
43	17·084	17·003	16·922	16·842	16·761	16·682	16·603	16·524	16·445	16·368
44	16·290	16·213	16·136	16·060	15·984	15·909	15·834	15·759	15·685	15·611
45	15·537	15·464	15·392	15·319	15·247	15·176	15·105	15·034	14·964	14·893
46	14·824	14·754	14·685	14·617	14·549	14·481	14·413	14·346	14·279	14·212
47	14·146	14·080	14·015	13·950	13·885	13·820	13·756	13·692	13·629	13·565
48	13·503	13·440	13·378	13·316	13·254	13·193	13·132	13·071	13·010	12·950
49	12·890	12·831	12·772	12·713	12·654	12·595	12·537	12·479	12·422	12·365
50	12·308	12·251	12·194	12·138	12·082	12·027	11·971	11·916	11·861	11·807
51	11·752	11·698	11·644	11·591	11·538	11·485	11·432	11·379	11·327	11·275
52	11·223	11·171	11·120	11·069	11·018	10·967	10·917	10·867	10·817	10·767
53	10·718	10·669	10·620	10·571	10·522	10·474	10·426	10·378	10·330	10·283
54	10·235	10·188	10·141	10·095	10·048	10·002	9·956	9·910	9·865	9·819
55	9·774	9·729	9·685	9·640	9·596	9·551	9·507	9·464	9·420	9·377
56	9·333	9·290	9·247	9·205	9·162	9·120	9·078	9·036	8·994	8·953
57	8·911	8·870	8·829	8·788	8·748	8·707	8·667	8·627	8·587	8·547
58	8·507	8·468	8·428	8·389	8·350	8·312	8·273	8·234	8·196	8·158
59	8·120	8·082	8·045	8·007	7·970	7·932	7·895	7·858	7·822	7·785
60	7·749	7·712	7·676	7·640	7·604	7·569	7·533	7·498	7·463	7·427
61	7·392	7·358	7·323	7·288	7·254	7·220	7·186	7·152	7·118	7·084
62	7·050	7·017	6·984	6·950	6·917	6·885	6·852	6·819	6·787	6·754
63	6·722	6·690	6·658	6·626	6·594	6·563	6·531	6·500	6·468	6·437
64	6·406	6·375	6·345	6·314	6·283	6·253	6·223	6·192	6·162	6·132
65	6·103	6·073	6·043	6·014	5·984	5·955	5·926	5·897	5·868	5·839
66	5·811	5·782	5·753	5·725	5·697	5·669	5·641	5·613	5·585	5·557
67	5·529	5·502	5·474	5·447	5·420	5·393	5·366	5·339	5·312	5·285
68	5·259	5·232	5·206	5·179	5·153	5·127	5·101	5·075	5·049	5·024
69	4·998	4·972	4·947	4·922	4·896	4·871	4·846	4·821	4·796	4·771

	00	10	20	30	40	50	60	70	80	90
7	4·747	4·504	4·270	4·044	3·826	3·616	3·412	3·216	3·026	2·870
8	2·664	2·491	2·324	2·163	2·006	1·854	1·707	1·564	1·425	1·291
9	1·160	1·034	·911	·791	·675	·562	·452	·345	·241	·140
10	·042	—	—	—	—	—	—	—	—	—

Table IV (*continued*).

PROPOSED STANDARD GLASS-WORKING TOOLS.

Curvatures in Dioptres.

	0	1	2	3	4	5	6	7	8	9
40	50·625	50·874	51·123	51·374	51·626	51·878	52·132	52·387	52·642	52·899
41	53·157	53·416	53·675	53·936	54·198	54·461	54·725	54·990	55·256	55·523
42	55·791	56·060	56·331	56·602	56·874	57·148	57·423	57·699	57·975	58·253
43	58·533	58·813	59·094	59·377	59·661	59·945	60·231	60·519	60·807	61·096
44	61·387	61·679	61·972	62·266	62·562	62·859	63·157	63·456	63·756	64·058
45	64·361	64·665	64·970	65·277	65·585	65·894	66·204	66·516	66·829	67·144
46	67·459	67·776	68·095	68·414	68·735	69·058	69·381	69·706	70·033	70·361
47	70·690	71·020	71·352	71·686	72·021	72·357	72·695	73·034	73·374	73·716
48	74·060	74·405	74·751	75·099	75·449	75·800	76·152	76·506	76·862	77·219
49	77·577	77·937	78·299	78·663	79·027	79·394	79·762	80·132	80·503	80·876
50	81·25	81·63	82·00	82·38	82·77	83·15	83·53	83·92	84·31	84·70
51	85·09	85·48	85·88	86·27	86·67	87·07	87·48	87·88	88·29	88·69
52	89·10	89·51	89·93	90·34	90·76	91·18	91·60	92·02	92·45	92·87
53	93·30	93·73	94·17	94·60	95·04	95·48	95·92	96·36	96·80	97·25
54	97·70	98·15	98·60	99·06	99·52	99·98	100·44	100·90	101·37	101·84
55	102·31	102·78	103·26	103·73	104·21	104·70	105·18	105·67	106·16	106·65
56	107·14	107·64	108·14	108·64	109·14	109·65	110·16	110·67	111·18	111·70
57	112·22	112·74	113·26	113·79	114·32	114·85	115·38	115·92	116·46	117·00
58	117·55	118·09	118·65	119·20	119·75	120·31	120·88	121·44	122·01	122·58
59	123·15	123·73	124·31	124·89	125·48	126·06	126·66	127·25	127·85	128·45
60	129·05	129·66	130·27	130·89	131·50	132·12	132·75	133·37	134·00	134·64
61	135·27	135·91	136·56	137·20	137·86	138·51	139·17	139·83	140·49	141·16
62	141·84	142·51	143·19	143·87	144·56	145·25	145·95	146·65	147·35	148·06
63	148·77	149·48	150·20	150·92	151·65	152·38	153·12	153·86	154·60	155·35
64	156·10	156·86	157·62	158·38	159·15	159·92	160·70	161·49	162·27	163·07
65	163·86	164·67	165·47	166·28	167·10	167·92	168·75	169·58	170·41	171·25
66	172·10	172·95	173·81	174·67	175·54	176·41	177·29	178·17	179·06	179·95
67	180·85	181·75	182·67	183·58	184·50	185·43	186·37	187·30	188·25	189·20
68	190·16	191·12	192·09	193·07	194·05	195·04	196·04	197·04	198·05	199·06
69	200·08	201·11	202·15	203·19	204·24	205·29	206·36	207·43	208·50	209·59

	00	10	20	30	40	50	60	70	80	90
7	210·68	222·02	234·18	247·25	261·34	276·55	293·04	310·96	330·51	351·91
8	375·43	401·41	430·23	462·39	498·51	539·34	585·87	639·37	701·52	774·59
9	861·72	967·37	1098·16	1264·20	1481·97	1779·99	2212·60	2897·43	4145·56	7136·54
10	23958·75	—	—	—	—	—	—	—	—	—

Table V.

PROPOSED STANDARD GLASS-WORKING TOOLS.

Radii in Inches.

	0	1	2	3	4	5	6	7	8	9
0	∞	852	424	282	211	168	140	119	104	92
1	82·7	74·9	68·4	63·0	58·3	54·2	50·7	47·5	44·7	42·2
2	40·01	37·98	36·14	34·46	32·92	31·51	30·20	28·99	27·87	26·83
3	25·85	24·94	24·09	23·29	22·53	21·82	21·15	20·52	19·92	19·35
4	18·81	18·30	17·81	17·34	16·90	16·47	16·07	15·68	15·31	14·95
5	14·61	14·28	13·97	13·67	13·37	13·09	12·82	12·56	12·31	12·07
6	11·83	11·61	11·39	11·17	10·97	10·77	10·58	10·39	10·21	10·03
7	9·861	9·696	9·535	9·378	9·226	9·078	8·934	8·793	8·657	8·525
8	8·395	8·269	8·146	8·026	7·909	7·795	7·684	7·575	7·469	7·365
9	7·264	7·165	7·068	6·974	6·881	6·791	6·702	6·616	6·531	6·448
10	6·367	6·287	6·209	6·133	6·058	5·985	5·913	5·843	5·774	5·706
11	5·639	5·574	5·510	5·447	5·386	5·325	5·266	5·207	5·150	5·094
12	5·038	4·984	4·931	4·878	4·826	4·776	4·726	4·677	4·629	4·581
13	4·534	4·488	4·443	4·399	4·355	4·312	4·270	4·228	4·187	4·146
14	4·106	4·067	4·029	3·990	3·953	3·916	3·879	3·844	3·808	3·773
15	3·739	3·705	3·672	3·639	3·606	3·574	3·542	3·511	3·481	3·450
16	3·420	3·391	3·362	3·333	3·304	3·276	3·249	3·222	3·195	3·168
17	3·142	3·116	3·090	3·065	3·040	3·015	2·991	2·967	2·943	2·920
18	2·897	2·874	2·851	2·829	2·806	2·785	2·763	2·742	2·721	2·700
19	2·679	2·659	2·639	2·619	2·599	2·580	2·560	2·541	2·522	2·504
20	2·4853	2·4671	2·4491	2·4313	2·4136	2·3962	2·3789	2·3618	2·3449	2·3282
21	2·3116	2·2952	2·2790	2·2630	2·2471	2·2314	2·2159	2·2005	2·1852	2·1701
22	2·1552	2·1404	2·1258	2·1113	2·0969	2·0827	2·0686	2·0547	2·0409	2·0272
23	2·0137	2·0003	1·9870	1·9739	1·9608	1·9479	1·9352	1·9225	1·9099	1·8975
24	1·8852	1·8730	1·8609	1·8489	1·8371	1·8253	1·8137	1·8021	1·7907	1·7793
25	1·7681	1·7570	1·7459	1·7350	1·7241	1·7134	1·7027	1·6921	1·6817	1·6713
26	1·6610	1·6508	1·6407	1·6306	1·6207	1·6108	1·6011	1·5914	1·5817	1·5722
27	1·5627	1·5534	1·5441	1·5349	1·5257	1·5166	1·5076	1·4987	1·4899	1·4811
28	1·4724	1·4637	1·4552	1·4467	1·4382	1·4298	1·4215	1·4133	1·4051	1·3970
29	1·3890	1·3810	1·3731	1·3652	1·3574	1·3497	1·3420	1·3344	1·3268	1·3193
30	1·3119	1·3045	1·2971	1·2899	1·2827	1·2755	1·2684	1·2613	1·2543	1·2473
31	1·2404	1·2336	1·2268	1·2200	1·2133	1·2066	1·2002	1·1934	1·1869	1·1805
32	1·1740	1·1677	1·1613	1·1550	1·1486	1·1426	1·1364	1·1303	1·1242	1·1182
33	1·1122	1·1063	1·1004	1·0945	1·0887	1·0829	1·0772	1·0715	1·0658	1·0602
34	1·0546	1·0490	1·0435	1·0381	1·0326	1·0272	1·0219	1·0165	1·0112	1·0060
35	1·0007	·9955	·9904	·9853	·9802	·9751	·9701	·9651	·9601	·9552
36	·9503	·9455	·9406	·9358	·9311	·9263	·9216	·9169	·9123	·9077
37	·9031	·8985	·8940	·8895	·8850	·8805	·8761	·8717	·8674	·8630
38	·8587	·8544	·8502	·8459	·8417	·8375	·8334	·8292	·8251	·8210
39	·8170	·8130	·8090	·8049	·8010	·7970	·7931	·7892	·7854	·7815
40	·7777	·7739	·7701	·7664	·7626	·7589	·7552	·7515	·7479	·7443
41	·7407	·7371	·7335	·7300	·7264	·7229	·7194	·7160	·7125	·7091
42	·7057	·7023	·6989	·6956	·6922	·6889	·6856	·6823	·6791	·6758
43	·6726	·6694	·6662	·6631	·6599	·6568	·6537	·6506	·6475	·6444
44	·6414	·6383	·6353	·6323	·6293	·6263	·6234	·6204	·6175	·6146
45	·6117	·6088	·6060	·6031	·6003	·5975	·5947	·5919	·5891	·5863
46	·5836	·5809	·5782	·5755	·5728	·5701	·5675	·5648	·5622	·5595
47	·5570	·5543	·5518	·5492	·5467	·5441	·5416	·5391	·5366	·5341
48	·5316	·5291	·5267	·5243	·5218	·5194	·5170	·5146	·5122	·5099
49	·5075	·5052	·5028	·5005	·4982	·4959	·4936	·4913	·4891	·4868
50	·4846	—	—	—	—	—	—	—	—	—

DISCUSSION*

Mr Horace Lee has communicated the following memorandum on a tool list, which was presented to the Optical Standards Committee in September 1918.

An attempt is here made to compile a formula on which a tool list could be built, so as to give *regular* differences between the tools. The requirements of such a formula are:

(1) It should be fairly simple.
(2) It should give a *finite* number of tools.
(3) The differences of radii should be such as are convenient in practice.

Considering first the simple progressions; the arithmetical progression—a constant difference of radii—is obviously unsuitable. The geometrical progression represents a constant percentage increase of radius. The connexion between r (radius) and n (number of the tool) is

$$r = a\,(1 + x/100)^{n-1},$$

where a is the first tool on the list and x is the percentage increase. In some ways this is ideal, for it would allow a lens of any focal length being made with equal accuracy; but a tool list compiled on these lines could never exist because it would be infinitely large. There remains the harmonic progression, or constant increase in curvature; this is suitable (and is used) for spectacle lens makers, where only the focal length of the lens has to be considered, but it is otherwise unsuitable, for suppose an interval of 10″ was suitable at 100″ radius, then this scheme would only require ·09″ interval at 10″ and ·0009″ at 1″. An attempt to combine geometric and harmonic progression leads practically to the same thing as geometric increase.

In seeking for other relationships between r and n one attempts to work upon the following considerations:

(1) Successive surfaces should not be "within colour," i.e., there should be a difference of at least ·0005″ in versine. This gives $\delta t = \cdot0005'' = \frac{1}{2}\left(\frac{h}{r}\right)^2 \delta r$, where h is the semi-diameter of the lens to be worked. In the case of an 8″ $f/4{\cdot}5$ lens, where the diameter is 1·8″, the deepest curve used would be about 2″. We then get the following intervals

$r =$	2″	$\delta r =$	·005
	10″		·124
	100″		12·4

A 4″ lens would, however, only require intervals four times as large. This rule of course imposes a lower limit to δr.

* All the following contributions have been received from members to whom advance proofs of the paper were sent.

(2) The accuracy with which tools can be made, in the ordinary way, to fit brass gauges. This of course varies with the workmen, but generally speaking an accuracy of ·001″ in radius is easily obtainable up to about 1″.

(3) If the accuracy of a model lens is to be reproduced in a lens of any scale, the tool list must be a percentage one. The most feasible thing in the case of photographic lenses is to make the tool list suitable for making models of a definite focal length (8″ is the usual size) and, when lenses of longer focus have to be reproduced, to be prepared to make special tools where necessary.

Let us see how that would affect other systems.

(a) *Microscope objectives*. The important curves are so short and the tools therefore small and easily made, that it is quite possible to make the exact tools required. The shallow curves are relatively insensitive and the accuracy of the main tool list would be quite sufficient

(b) *Eye-pieces*. Very great accuracy is not necessary here, so that these will not be troublesome.

(c) *Small telescope object glasses*. The curves are relatively shallow to aperture, so that considerations of colour (1) allow fairly large intervals—larger than in the lists given below. Take the binocular object glass with diameter ·95″. The deepest curve is about 2″.

$$\text{From (1)} \quad \delta r = \cdot 016'' \quad \text{for} \quad r = 2''$$
$$\text{and} \qquad \delta r = \cdot 4'' \qquad \text{for} \quad r = 10''.$$

(d) *Large telescope object glasses*, where great accuracy is required. Here considerations of glass (see below) are relatively more important and make a certain amount of empiricism necessary.

(4) Too high an accuracy in radius is thrown away on account of the uncertainty prevailing in glass readings. The guaranteed accuracy is not more than ·001 in refractive indices between two meltings, corresponding to ·2 per cent. error in radius.

(5) A practical example will form a useful guide. An alteration to a lens was made. With the different glass used it was necessary to alter a curve from 1·275 to 1·269. Calculation showed that 1·275 would affect the corrections. This points to the fact that ·012″ is too large an interval in the neighbourhood of 1″ radius.

(6) One need only consider tools greater than about ·5″, as smaller ones can be quickly made as required.

From these considerations differential equations were formed and the constants chosen to give suitable intervals at chosen values of r. These can be formed in any way to suit prescribed conditions, consistent with ease in integration and simplicity in the resulting formula.

Examples

$$dr/dn = \cdot 0014 r^{\frac{1}{2}} + \cdot 0059 r^{\frac{3}{2}},$$

whence

$$n + 555 \cdot 6 = 696 \tan^{-1} (2 \cdot 053 r^{\frac{1}{2}}).$$

This formula gives the following scale (starting from $r = \cdot25$):

Radius	n	Interval between the tools	Interval between radii	Number of tools in intervals by formula	Number of tools in intervals by T.T.H. list
·25″	0	·00144			
·5	117·5	·00308	·25– ·5	117·5	23
1	222·2	·0073	·5 –1·0	104·7	70
2	306·4	·0186	1·0 –2·0	84·2	70
3	347·0	·0334	2–3	40·6	36
4	371·4	·050	3–4	24·4	25
5	388·4	·069	4–5	17·0	18
10	431·4	·191	5–10	43·0	46
20	462·1	·534	10–20	30·7	26
50	489·8	2.1	20–50	27·7	21
100	503·8	5·91	50–100	14·0	8
400	521·4	47·2	100–400	17·6	10
∞	537·6	∞	400– ∞	16·2	1

Commenting on the curves given by the various formulae, Mr Lee says: The following is a comparison between my lists and yours:

Radius	Interval between tools		
	T.S.	H.L. (1)	H.L. (2)
·25″	·00118	·00144	·00106
·5	·0023	·0031	·0023
1	·0052	·0073	·0054
2	·0133	·0186	—
3	·024	·0334	—
4	·039	·050	·0393
5	·054	·069	·056
10	·229	·191	·180
20	·60	·534	·522
50	3·2	2·1	3·49
100	12·0	5·91	12·7
No. of tools ·5 to ∞	494	420	(?)

You will see that my second list gives intervals very similar to those given by your list. The differential equation, however,

$$dr/dn = \cdot00022r^{\frac{1}{2}} + \cdot00392r^{\frac{3}{2}} + \cdot001268r^2$$

is very awkward and would need solving graphically, or the tool list could be built up from the values of dr. Thus ($R = 1/r$)

$$\frac{dR}{dn} = \frac{\cdot00022}{r^{\frac{1}{2}}} + \frac{\cdot00392}{r^{\frac{3}{2}}} + \cdot001268.$$

The first 20 tools are:

∞	787	392	261	195	156	132	111	97	86
77·3	70·2	64·2	59·1	54·7	51	47·7	44·8	42·2	39·9

Mr J. W. French: It would certainly be a great convenience to the manufacturer if he could obtain from stock the standard test plates and tools that he

requires. It seems to me that the question of standard test plates is of most importance because the installation of suitable machines for turning metal tools is not a matter of serious difficulty to a firm of moderate size. There are, however, many small firms who do find difficulty in obtaining the tools they require, especially at the present time.

So far as our curved work experience goes—that is, so far as objectives and eyepieces of all kinds are concerned—the steps from ten inches downwards appear to be unduly small. Our practice suggests steps of about twice the width, but you will understand that I can offer no opinion as regards the small curves of microscope objectives.

From the commercial point of view, the number of standards should not be greater than is actually necessary.

Above ten inches the steps proposed by you are in fair agreement with our practice and with the curve indicated by circles. Below four inches our practice is more nearly in agreement with the latter curve than with yours. Unless, therefore, it is essential from the point of view of the microscope objective maker, there is reason to hope that your range of one thousand curves might be considerably reduced.

Is there any particular reason why the millimetre radius table should be more extended than the inch table? The latter is terminated at a radius of 0·4846″, that is 12·3 mm., whereas the former is extended to 0·042 mm. In any case a small radius of this latter order appears to me to be quite impracticable.

With regard to the inch radii table, the fourth decimal figure might well be omitted and at the longer focus end the third figure. We cannot in routine practice measure even the small tool radii to anything approaching one ten-thousandth of an inch. Five thousandths is a more practicable figure. At the long focus end the second decimal figure might be dispensed with, as it is often possible to adjust the computational data to suit standard tools.

Thus in these respects it seems to be possible to reduce the list very appreciably, which I think is desirable if the scheme is to be successful.

I fear, however, that the list would require to be extended owing to the need for series of test plates of the same curvature but of different diameters. In high class work a one-inch diameter test plate is not suitable for a four or five-inch diameter objective. It is desirable in testing figure that the test plate should take in nearly the whole surface. On the other hand a four-inch test plate is liable to rock when applied to a one-inch surface, and in any case to adopt an expensive four or five-inch test plate for a one-inch surface is undesirable.

Mr Conrad Beck: It is my opinion that standardisation may be an advantage or a disadvantage according to circumstances. On the one hand it may be a convenience to the consumer, or a means of reducing cost or improving the quality to the manufacturer. On the other hand it has a great tendency to hamper advance if it stereotypes models, or it may produce bad instruments if it induces the construction of instruments out of standard parts which are not essentially suited to work in combination. There is no virtue in standardisation itself apart from circumstances.

The question of standardising glass grinding tools has only advantages if a central supply for supplying manufacturers is desirable. It is very doubtful if any considerable economy to the manufacturer would be created by such a supply, and I am of the opinion that with the inevitable variations that must exist in the refraction of optical glass it is most unlikely that the makers of the best instruments would limit their tools to standard curves.

I look with grave suspicion on any attempt to standardise curves or foci of optical elements as tending to give the impression that lenses corrected for one set of conditions can be used for any other, calling for a similar focal length.

Mr F. Twyman, in response to a suggestion that experience already gained by the use of the lens interferometer may throw light on the validity of some of the assumptions made in the paper, has communicated replies to two questions.

1. Should the standard of definition be defined in local differences of phase, or on the area of the image patch?

My reply to this is that a precise and complete knowledge of the distribution of intensity in the region of the geometrical image is only obtainable on knowing the aberrations of phase in the emergent beam. Thus no standard of definition is likely to be permanently satisfactory unless it is based on phase aberrations. But it must be admitted that no practical and simple method of predicting from the aberrations of phase in the emergent beam the degree of definition of the optical system is yet available. One is probably justified in saying that unless the aberrations are smaller than 1/4 wave length, the image is recognisably not perfect (Lord Rayleigh's rule). But to derive a numerical statement of want of definition from any particular distribution of phase aberration entails a complete knowledge of the relation between aberrations and distribution of intensity of illumination in the region of the focus; a knowledge which up till now has not been forthcoming in any form which could be readily utilised. It was my feeling of the importance of improving our knowledge of this relation that caused me to lay the problem before Dr Silberstein, whose endeavour to fill this hiatus more satisfactorily than heretofore was partly published in his paper in the *Phil. Mag.* vol. xxxv. p. 30. It seems to me quite certain that to attempt to base a standard of definition in terms of the area of the image patch at any particular plane near the "focus" is unsatisfactory for the following reason:

The knowledge of the definition of the lens must be incomplete unless the distribution of intensity is known; and this not only throughout the area of the image patch, but at every point of space in the vicinity of the focus.

2. Is the wave-length standard in my opinion realisable in large aperture photographic lenses of simple construction when the glass is homogeneous, and the surfaces are strictly spherical?

I am unable to answer this question. Owing to the necessity of confining our attention recently to immediately productive work, we have had to defer for the time being the programme of research on photographic lenses which we had projected. All I can say is that we have hitherto found no large aperture photographic lenses which even approximately realise the wave-length standard. Even

for axial pencils the errors in small $f/3{\cdot}5$ lenses usually amount, after single transmission through the lens, to 2 wave-lengths; while for the extremity of the photographic plate the average error amounts to 14 wave-lengths. Our experience is derived from lenses by a number of the best known British and German makers.

I think that were the computer to derive assistance in his work from interferometer tests, he might be able to improve considerably on the above. But I must say I have never considered it likely that with strictly spherical surfaces and homogeneous glass, it would ever be possible to manufacture photographic lenses whose phase aberrations should be confined within the limits of Lord Rayleigh's rule.

Mr H. Dennis Taylor: It seems to me that Mr Smith's suggestion contained in the last paragraph but one on page 250, concerning the possibility of some firm taking up tool making for the trade as a special branch of manufacture, is a very good one. Of course it could not be expected that optical firms, when starting operations, should lay themselves out to make a system of tools of the right spacing. What they generally do is to make tools to certain sets of radii and curves which are already specified, and to have arrived at a complete series of properly spaced tools from the start would imply an unusual degree of foresight. It is curious that Mr Smith's Table V gives a number of tools of radii both above and below 2 inches which are only slightly in excess of the actual number used in our own workshops, but our spacings are not so regular and systematic.

I would like to point out, however, that radii such as 852, 424, 282, 211, 168, 140, 119, 104 and 92, etc., want two or three more tools at least between each pair, before they are graduated nicely enough for the very best results in large telescope objectives, and, as a result of this, we find our long radius tools are more closely spaced than in Mr Smith's projected series and our short radius tools less closely spaced.

In spite of this, when we are giving the final chromatic correction to large objectives and especially triple or photo-visual objectives we have frequently to alter a pair of tools to an intermediate position between itself and a neighbour, an operation that usually takes much less time than making a new pair of tools.

It is to be hoped that some firm will lay themselves out to manufacture for the trade spherical tools according to a plan substantially like Mr Smith's. The best relative diameters for the tools remain to be considered.

Mr H. L. Aldis: The general scheme set out in the paper seems to me quite sound and reasonable, but I would suggest that if the scheme is to take a practical form it would have to be in the nature of the establishment of a set of standard proof glasses.

It would undoubtedly be a convenience to opticians to be able to obtain replicas of proof glasses of a definite radius as the question of supplying tools to fit these would not be a serious matter, the chief difficulty in making any new lens or series of lenses being the question of making proof glasses of the requisite radii. The spacing you have given I should say would be quite sufficient for all ordinary purposes.

On the question of the commercial aspect of the standardisation of proof glasses in accordance with this scheme I find it rather difficult to express an opinion. The proof glasses would have to be made with great accuracy and the expense would undoubtedly be very great, possibly £10 for each proof glass, so that it is difficult to say whether the demand for replicas of these proof glasses would be sufficient to recoup the capital expenditure that would be required to make such a large number as 1000 of these as suggested in your paper.

I have gone roughly through your list and compared it with the actual set of proof glasses which form our stock-in-trade. Apparently the steps suggested by your paper would give three steps on an average between the proof glasses which we employ, but our stock of proof glasses is certainly not sufficient for the purpose that we need them for and we are continually having to add to our list.

Mr H. A. Hughes: We consider it a very wasteful practice that at the present time optical tools are made up individually. If produced in quantity it would pay to construct special machinery and enable glass shops to be equipped with a series of a sufficiently wide range.

We prefer to work our tools in millimetres, but are quite willing to fall into line if it is decided to use decimals of an inch. We are constantly being hindered in our work for want of tools.

I notice you also mention test plates, but I think these are best done in the shop, as each manufacturer would prefer to make his own.

I hope very much that as a result of your paper a meeting of the whole Trade will be held and a decision come to.

In reply the author desires to express his thanks to all the contributors for their very interesting communications. In particular he would like to thank Mr Lee for permitting his investigation to be published as part of the discussion.

Mr Beck has raised the question whether standardisation of tools would prove advantageous. This is undoubtedly the most important question which arises for decision, and that decision can only be made by those who are alone affected by it, the manufacturers themselves. From the discussion and other evidence it is however clear that a simple formula can be found which gives tools regularly spaced over the whole possible range, and with intervals in remarkably close agreement with those which various manufacturers have found necessary in their experience of lens making extending over many years. The way in which such a formula rests on a theoretical foundation renders it highly probable that it would fit in, to the same degree of accuracy, with any new developments in instrument construction, however unforseen their character, provided that these do not involve the abandonment of spherical surfaces. If therefore tools were manufactured on such a basis there is little room for fear that progress would be retarded thereby. The main question is whether the possibility of obtaining test plates and tools of reliable accuracy from a central source would prove of such advantage to a sufficiently important group of manufacturers as to justify its realisation. It may be as well to point out that the standardisation of tools need by no means imply

the exclusion of curves not recognised under the system. The tools to be manufactured as standards would probably only include those members of any system for which there is at the time a fairly wide demand. The convenience of arranging other tools, when required, in accordance with a definite system is clear, but it may occasionally, though rarely, be found that a particular radius is required within closer limits than any reasonable scheme for general use would involve. As an example it is possible to imagine a machine for very accurate measurements in which the attainment of a definite curvature within the smallest possible limits would be a great convenience. Such cases are not considered in a general scheme of standardisation.

As far as I am aware the adoption of standard curvatures would occupy a unique place among schemes of standardisation, since the aim is not to secure interchangeability. No reasonable person would propose to cement a lens from one maker to one from another. There is also a wide gulf between the standardisation of a system of tools and the specification of the curves to be used in an optical element. The curves of any lens must of course be varied to suit the inevitable variations in the properties of the glass employed, but it is quite possible to satisfy these conditions utilising only the tools of a regularly spaced series.

The most general criticism that has been made relates to the number of tools included in the list in the neighbourhood of 1 cm. compared with those of long radius, and the suggestions made are equivalent to a proposal to substitute another formula giving approximately the same spacing for tools of more than 5 cm. radius, but reducing the number of the shorter tools. Such a list could very easily be constructed, but the evidence at present available shows that the resulting intervals would be too coarse for the microscope maker. The lists of manufacturers appear to be roughly divisible into two classes, typified by the curves of figure 2 represented by circles and crosses respectively. In the former and more frequent type the intervals are greater than in the illustrative list for radii below 5 cms., and tend to become definitely smaller for the long radii. In the other class on the other hand the intervals are decidedly greater for the long radii, but below about 8 cms. there are strong indications that intervals of about half the tabulated values are required. These smaller intervals in the case illustrated amount to more than ten per cent. of the whole number, and are almost uniformly distributed over the whole range from the smallest radius up to 8 cms. This reduced interval corresponds closely with the requirements mentioned by Mr Taylor for long radii. It would therefore appear that one end of the scale has not been more generously treated in the distribution of the tools than the other. That such a result should have been reached in the first numerical case tried is no doubt largely an accident, but it tends to show that the basis on which the list is built is not unreasonable.

Considerations of a very general and simple character suffice to show that the tools of longest radius for the telescope maker should form an arithmetical progression of curvatures with zero as its starting point, and similarly that at the other end of the scale the microscope maker requires an approximately arithmetical progression of radii. In the latter instance the extreme members of a complete

series cannot be used, as has been pointed out by Mr French, but the common difference which the radii tend to have at this end of a series may nevertheless be conveniently denoted by the radius of the last unattainable member of the theoretical system. It is very easy to write down a formula for a list satisfying these conditions with assigned values for the common differences of the two end series and with its middle member having any arbitrarily assigned radius. Let such a system consist of $2N$ tools, with

$$r_0 = \infty, \quad r_2 = \tfrac{1}{2}r_1, \quad r_3 = \tfrac{1}{3}r_1, \ldots \text{ approximately}$$

and $$r_{2N} = 0, \quad r_{2N-2} = 2r_{2N-1}, \quad r_{2N-3} = 3r_{2N-1}, \ldots \text{ approximately},$$

the values of r_1, r_N, r_{2N-1} being assigned arbitrarily. The simplest formula satisfying these requirements is

$$r_n = -\frac{2N-n}{n} \cdot \frac{r_N r_{2N-1}(2N-1)^2(n-1) + 2r_1 r_{2N-1}(2N-1)(N-n) - r_1 r_N (2N-1-n)}{r_1(n-1) + 2r_N(2N-1)(N-n) - r_{2N-1}(2N-1)^2(2N-1-n)}$$

and the distribution it gives resembles very closely that of any other reasonable formula satisfying the same conditions. By making a suitable choice of r_N the peak of the resulting distribution curve can be given any required position, and a formula thus obtained for such a list as has been virtually suggested in the discussion. This formula takes the simpler shape

$$r_n = \frac{2N-n}{n} r_N$$

if $$r_1 = (2N-1)r_N = (2N-1)^2 r_{2N-1}$$

or apart from the precise value given to N if

$$r_N^2 = r_1 . r_{2N-1}.$$

Now it has been seen that the existing list errs about equally in the spacing of both the very long and the very short radii, so that if both are treated equally generously and this simplified formula is used r_N remains fixed, its value being approximately 14·7 mm. This simplified formula however is a special case of the more general one considered in the paper, α and β being given equal values. Equations (7), (9) and (23) show that it corresponds to the assumption that the ratio of the shortest radius to the aperture will not change as the absolute value of the aperture varies, an assumption which is certainly untenable. It follows that the radius of the central tool may not be as great as 14·7 mm., a still greater value being excluded *a fortiori*. The centre of gravity of a system of intervals covering the requirements of the whole industry must therefore lie approximately where it is placed in the illustrative list, and such a list provides a suitable basis for the numbering of tools. The definite adoption of such a list for standard purposes was suggested because the intervals appear to be approximately of the correct order of magnitude.

The manufacturer of large telescope objectives will probably find it necessary to introduce half or quarter numbers for a few of the long radius tools. In other

cases it should be possible to take advantage of the fact that the list is not a geometrical series to avoid the need for intermediate numbers by adjusting slightly the focal length of the lens. This is in accordance with the principle advocated by Mr French of keeping down the number of tools as far as possible.

The only logical ground on which a movement of the centre of gravity of a universal tool list towards longer radii could be based is the demonstration that a small alteration of radius produces more serious changes in aberrations which do not appear in the centre of the field than in those which affect the whole field equally. The theory of aberrations of systems composed of aspherical surfaces does not render this likely, as in compact systems the departure from the sphere affects central aberrations of a lower order than the lowest orders of oblique aberrations which it influences.

With reference to particular questions raised by Mr French, the inch table is a simple conversion of the mm. table. The smaller unit is more suitable for small microscope work and seems to be extensively used. There appeared therefore to be no great gain in continuing the inch table beyond the ordinary range for telescope eyepieces. Except for very small radii the table of curvatures is the most convenient for optical calculations, the other tables being more suitable as a rule for the tool maker.

As regards the curtailment of the number of figures, enough have been retained to specify the interval to at least one part in twenty. In a number of cases no great value accordingly attaches to the last digit, but the line is drawn in a fairly convenient place. Mr French's remarks on the accuracy of measurement of curvatures are surprising. The uncertainty is many times as large as would be given by a reasonable spherometer. There is no reason why the accuracy signified in the tables should not in general be realised if it were called for. In our experience small radii can be measured without great difficulty to an accuracy of one part in five thousand, and if necessary this figure could be reduced. Presumably in the work in which Mr French has been engaged no great accuracy in the tools is called for, and rough means of measuring them have proved sufficiently accurate. In this connexion it may not be without interest to mention that spherical ended gauges can be measured to a higher accuracy than one part in one hundred thousand. This indicates that appreciably higher accuracy in the determination of curvatures than is now attained is possible.

Mr Twyman's statement on the correct criterion for the standard of definition in lenses is of great interest, and it is to be hoped that further progress in the work he mentions will be made in the near future. It would not be surprising to find that the difference in the results given by a wave theory and a ray theory of image formation only become of importance when a standard of definition higher than is now usual in photographic lenses is attained. In the author's opinion the view given by Mr Twyman in his last sentence is likely to prove well founded.

It is very interesting to find that Mr Lee's second formula gives results so closely in agreement with those of the other paper. At the short end of the scale the results differ, those by Mr Lee's formula being unsuitably spaced. This

formula will therefore be less well adapted for the construction of a complete system.

The discussion has served the purpose of showing that there is a sufficiently strong prima facie case for the standardisation of tools for the subject to receive careful consideration from the industry as a whole. If a decision to proceed on the general question were reached a number of matters not touched on in this paper would come up for settlement, notably that of the diameters of tools and test plates mentioned by Mr Taylor and others. These are questions which can only be decided in the light of manufacturing experience. Should it be considered that members of the staff of the National Physical Laboratory can assist in any measures which the industry conclude would be for their benefit, their readiness to co-operate can be relied on.

ABSTRACTS

"How to Cut Glass Properly." *Hardware and Metal.* [Abstr. from *Scientific American Supplement*, 87 (8th March, 1919), 151.]

There is much loss in the course of a year through the breakage of glass. It is one of the serious "leaks" in the hardware business.

Sheet glass should always be cut on the hollow or concave side. One can easily tell the concave side by holding the glass up and then glancing down the edge. This slightly hollow or concave nature of the glass is caused through the process of manufacture.

In the manufacture of sheet glass it is blown into cylindrical form that resembles a huge bottle. This great bottle-like structure of glass, which is from 15 to 20 ft in length and about 3 ft in diameter, is cut into sections approximating 5 ft in length. These sections are split open at one point by drawing a hot iron across. They are then laid on a mould and flattened by means of heat from a coke furnace. It is never possible to make them exactly flat. The sheets are slightly concave.

There is greater resistance on the convex side than on the concave side. This accounts for the cut "running out," and invariably a spoiled sheet if the cut is made along the convex side. It is essential to safeguard against waste that the glass be cut along the hollow concave side. But it is also very necessary that the cutting tool be kept in good condition. Care should be taken that it does not get dull, or that the bearing in which it runs does not become worn. If the cutter becomes dull it will skip in places. These little skips are danger places that often cause the cut to run out. If a wheel cutter is used the little wheel should be kept oiled. When not in use if it is kept covered in coal oil it will always be in good shape. As soon as it begins to show signs of dullness it should be discarded or the wheel should be replaced. Where a diamond is used it is always advisable for only one person to use the diamond. It is seldom that two people hold a glass-cutter in the same manner. Every clerk who is allowed to cut glass should have a cutter or diamond for his own use. And he should take good care of it. The breakage of two or three large lights of glass will cause a loss in actual money, equal to the cost of a good many glass-cutters.

"Experiments with Photo-electric Cells." By E. Barkow, *Phys. Z.* 18 (1917), 214.

The extensive use of the Elster and Geitel photo-electric cells has been largely due to the fact that over considerable ranges the strength of the photo-electric current excited in them has been found to be strictly proportional to the illumination, and further that the cells are free from disturbances due to fatigue. The author, however, in experiments with potassium cells of different makes found considerable variations in their behaviour when using high intensities. In the region between 47 and 1200 metre-candles the current increased more slowly than the intensity of the light in the case of two cells and more quickly in the case of another cell. The relation between current and intensity was found to depend on the potential difference applied to the electrodes of the cells. For one particular value of this potential difference another cell exhibited strict proportionality

between current and intensity over the same range. The character of the curves connecting current and potential difference was unaltered by variation of intensity, but was affected by want of uniformity in the distance between the two electrodes.

If the cells were subjected to strong illumination for lengthy periods broken by longer intervals of rest, the sensitiveness decreased in some cases by as much as 10 per cent. and then slowly increased again. The decrease in sensitiveness varied with different cells and seemed to have no definite relation to the length of the intervals of darkness.

These results are explained by the assumption that in the cell two distinct processes are going on, namely, a more sensitive substance is formed under the influence of the illumination, is further built up during the period of darkness, and finally breaks down, while on the other hand another combination is formed which breaks down under the influence of light. The author also suggests that there may be a variation in the size of the colloidal potassium particles when strongly illuminated.

The result of these observations is to show that the cells are not suitable for measurements of the sun's intensity, where an accuracy of 1 per cent. is required, unless the intensity of illumination is reduced by means of a rotating sector or some similar device. The disturbances noted do not seem to exist in the case of measurements of low and very low intensities, e.g. in the case of determinations of the intensities of double stars.

J. S. A.

"Experiments with Photo-electric Cells." By C. DORNO, *Phys. Z.* 18 (1917), 381.

As the result of measurements of sun and sky radiation, carried out at the Davos Observatory, the author comes to the conclusion that photo-electric cells are reliable for such measurements, provided that they are not subjected to too high intensities. His experiments are of interest in connection with the results mentioned in the previous abstract. For the measurement of sun's radiation two highly evacuated potassium cells were used, while for the sky radiation measurements two similar cells filled with argon were employed. The maximum current in the latter was $3{\cdot}5 \times 10^{-9}$ amp. and in the former 5×10^{-8} amp., in which case the duration of exposure was only for a few seconds. The currents were observed by means of galvanometers. In order to be independent of the varying sensitivity of the cells to light of different wave-lengths, filters were almost always employed; these were as follows: a 5 mm. strong Jenens blue uviol glass F 3873 transmitting between 430 and 390 $\mu\mu$, a 3 mm. strong blue uviol glass F 3653 transmitting between 430 and 240 $\mu\mu$, a yellow filter F 5899 as used by Voege for the determination of the physiologically active rays, a green filter as used by L. Weber in photometry, a net filter for cutting down the total radiation when the altitude of the sun is high, and later a 5 mm. thick yellow filter F 8713 Schott which absorbs the extreme blue and the whole of the ultra-violet. No filter was employed when the altitude of the sun was low.

The advantages of the vacuum cells are as follows: (1) they are insensitive to temporary overloads, (2) they are independent of the voltage, so that one can always work with saturated currents, (3) they give a greater photo-electric current, and (4) they do not vary with temperature so much as gas-filled cells do. It was found that the variations during a set of experiments lasting about 2 or 3 minutes were always less than 1 per cent.; the second reading was always greater when the sun was ascending, and less when it

was descending, as would be expected. Sudden variations in sensitivity and fatigue effects, such as were observed by Barkow, were not met with. The observed variations were probably due to haze in front of the sun, valley mists, or chimney smoke.

The proportionality of the photo-electric current to the intensity of the light was proved by observations on the sun's radiation on cloudless days and also by means of small glow lamps (2·48 volts, ·952 amp.). The form of the characteristics of the cells and also the absolute values were found to remain constant. In the case of an argon-filled cell the sensitivity increased by 4 per cent., but then remained constant.

It was found that the vacuum cells must be allowed to "age" for some months before they become constant. The cells must also be protected from heat radiation effects, otherwise the potassium will evaporate and cause great variations in the sensitivity.

The author comes to the conclusion that even in the case of the argon-filled cells the variations in sensitivity can with care be kept small. J. S. A.

"The Limit of Total Reflection." By Prof. HUGO KRÜSS, *Z. f. Instrk.* 39 (1919), 73.

The author discusses the question of the distribution of intensity in the neighbourhood of the critical angle in the case of light reflected and refracted at the boundary plane between two media of different refractive index. The problem has an important bearing on the accuracy of measurements with instruments, such as refractometers, which depend on the determination of critical angles. In refractometric measurements if the intensity falls off comparatively slowly at the critical angle, the estimate of that angle will be too low, so that the measured emergent angle will be too great, resulting in too small a value of the refractive index of the substance under examination. The author finds that for an angle 1′ less than the critical angle the intensity of the refracted light is still 65–94 per cent. of the intensity of the incident light in the case of relative refractive indices varying from 1·05 to 1·60. He comes to the conclusion that the critical angle will be defined by a very sharp line, a possible error of 1′ being out of the question. In measurements with the Pulfrich refractometer an error of 1′ of angle corresponds approximately to an error of 1 in the 4th decimal place of refractive index. One very often (in the case of testing glass specimens, for example) wishes to obtain indices correct to 1 or 2 units in the 5th decimal place. This corresponds approximately to ·1′ of angle, so that in very accurate refractometric work what should be considered is the variation of intensity *within* the first minute of angle inside the critical angle. The author only considers the intensity variation at 1′, 5′, etc., and bases his conclusions on the effect this variation may have on measurements of refractive index to 4th decimal place accuracy. J. S. A.

LIST OF RECENT BOOKS AND CURRENT SCIENTIFIC PAPERS IN OPTICS

Books.

Mirrors, Prisms, and Lenses. A text-book of Geometrical Optics. By Prof. J. P. C. Southall. (New York: The Macmillan Co.; London: Macmillan & Co., Ltd, 1918.) 3.25 dollars.

Simplified Method of Tracing Rays through any Optical System of Lenses, Prisms, and Mirrors. By Dr Ludwik Silberstein. With diagrams. Pp. ix + 37. (London: Longmans, Green & Co., 1918.) 5*s*. net.

Elements of the Electromagnetic Theory of Light. By Ludwik Silberstein, Ph.D. Pp. vii + 48. (London: Longmans, Green & Co., 1918.) 3*s*. 6*d*. net.

Displacement Interferometry by the Aid of the Achromatic Fringes. By Prof. Carl Barus. Pt III. Pp. 100. (Washington: The Carnegie Institution of Washington.)

Colour in Relation to Chemical Constitution [Monographs on Industrial Chemistry: Edited by Sir E. Thorpe]. By E. R. Watson, M.A., D.Sc. Pp. xii + 197, with 4 coloured plates and 65 figures of absorption curves, spectra, etc. (London: Longmans, Green & Co., 1918.) 12*s*. 6*d*. net.

Tables of Refractive Indices. Vol. I, Essential Oils. Compiled by R. Kanthack. Edited by Dr J. N. Goldsmith. Pp. 148. (London: Adam Hilger, Ltd, 1918.) 15*s*. net.

The British Journal Photographic Almanac and Photographer's Daily Companion, 1919. Edited by E. Brown. Pp. 644. (London: H. Greenwood & Co., Ltd, 1919.) 1*s*. 6*d*. net.

Die optischen Instrumente (Lupe, Mikroskop, Fernrohr, photographisches Objectiv und ihnen verwandte Instrumente). By Moritz v. Rohr. (From *Natur und Geisteswelt*, vol. 88.) 3rd edition. Pp. vi + 137, with 89 illustrations. (Leipsic and Berlin: B. G. Teubner, 1918.) 1.20 Marks, bound 1.50 Marks.

Das Auge und die Brille. By M. v. Rohr. 2nd edition. Pp. 105, with 84 illustrations and 1 plate. (Leipsic, 1918.) Bound, 1.80 Marks.

Geometrical Optics.

1. Panoramic Photographs and Perspective. By C. J. Stokes. Brit. Journ. Photography, 66 (14th Feb. 1919), 67.
2. The Limit of Total Reflection. By Prof. Hugo Krüss. Z. f. Instrk. 39 (Mar. 1919), 73. [Abstr. on p. 272 of this No. of Trans. Opt. Soc.]
3. The field of View: the Apparent Enlargement of the Moon, etc. on the Horizon, Parts II, III, and IV. By Hans Witte. Phys. Z. 20 (Feb., Mar. 1919), 61, 114, 126. [Part I, Phys. Z. 19 (1918), 142.]

Instrumental Optics.

4. British and German Lenses. Brit. Journ. Photography, 66 (3rd Jan. 1919), 2.
5. Aeroplane Photography. By Arthur Brocker, Jun. and L. J. R. Holst. (Reprint from "Aviation") Brit. Journ. Photography, 66 (21st Feb. 1919), 84.

6. Improvements in Lenses for Aerial Photography. By W. P. APPLETON. Photographic Journ. 59 (Apr. 1919), 114.
7. Development in Photographic Lenses for Aircraft. By J. HASSELKUS. Photographic Journ. 59 (Apr. 1919), 121.
8. Large Anastigmats for Aerial Photography introduced during the War by Messrs Aldis Brothers. Photographic Journ. 59 (Apr. 1919), 122.
9. New Telescopes. By H. S. J. Month. Notices, R.A.S. 79 (Feb. 1919), 299.
10. Photographic Gun. Aerial Age Weekly (7th Apr. 1919).
11. On a Prismatic Astrolabe for the Measurement of Variation of Latitude. By M. RENÉ BAILLAUD. Compt. Rend. 168 (Jan. 1919), 85.
12. New Projection Apparatus for Lecture Purposes. By J. D. La Nature, No. 2354 (22nd Mar. 1919), 303.
13. On Errors of Measurement in Single-Station Range-finders, I. By Dr HANS SCHULZ, Z. f. Instrk. 39 (Mar. 1919), 91.
14. Dr Koller's Tachymeter Construction. By TH. DOKULIL. Z. f. Feinmechanik, **27** (Feb., Mar. 1919), 25, 33.

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15. Optical Glass. Nature (27th Mar. 1919), 65.
16. Machines and Tools for Optical Purposes, III. By M. BLASCHKE. D. Opt. Wochenschr. (1919), 2.
17. The Nomenclature of Optical Glass. By W. ZSCHOKKE. D. Opt. Wochenschr. (1919), 30; also Z. f. Instrk. 38 (1918), 49. [Abstr. in Trans. Opt. Soc. 20 (1919), 122.]

OPHTHALMOLOGICAL OPTICS.

18. An Exhibition of Some Experiments on Colour Blindness. By C. R. GIBSON, F.R.S.E. Proc. Phys. Soc. 31 (Feb. 1919), 79.
19. Vision in Light and Darkness (Conclusion). By FELIX LINKE. D. Opt. Wochenschr. (1919), 4.
20. On the Purkinje Phenomenon. By FRANZ EXNER. Wien. Anz. (1919), 2.

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(*a*) *General.*

21. On Sommerfield's Treatment of the Problem of Diffraction by a Semi-infinite Screen. By SISIR KUMAR MITRA. Phil. Mag. 37 (Jan. 1919), 50.
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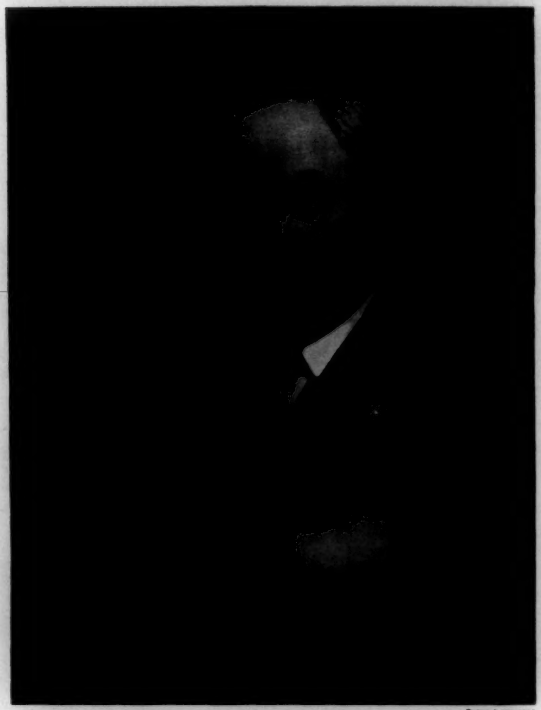
Patents.

British: see *Brit. Journ. Photography; Optician; Photographic Dealer.*

German: see *Z. d. D. Ges. f. Mech. u. Opt.; Z. f. Feinmech.*



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